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Technical Note N-402

WAVE-INDUCED MOTIONS OF A ROCKET VEHICLE
DRIFTING IN A VERTICAL ATTITUDE

by

J. J. Leendertse, P.E.

31 January 1961

62-1500

U. S. NAVAL CIVIL ENGINEERING LABORATORY
PORT HUENEME, CALIFORNIA

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Type C
Final Report

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OBJECT OF PROJECT

To determine the wave-induced motions of a large solid-propellant rocket vehicle drifting in a vertical attitude in wave environments described by Hydrographic Office sea states ranging from zero to five.

ABSTRACT

Results are presented of a theoretical and experimental study in the laboratory of the movements of a large solid-propellant rocket vehicle drifting in a vertical attitude in uniform waves and in the wave environment of the open sea. Measurements were made of the movements in heave and pitch of a 1-to-120 scale model of a million-pound vehicle, 175 feet long and about 12-1/2 feet in diameter. Heave measurements agree well with those calculated; no pitch calculations were made.

It appears that heave and pitch are linear functions of wave height and nonlinear functions of wave period. In deep water the ratio of heave to wave height is found to be 0.1 for waves with a 6-second period, and 0.4 for waves with a 12-second period. The maximum deviation from the vertical is found to be 9 minutes of arc for waves one foot in height with a 6-second period, and 11 minutes for waves one foot in height with a 12-second period.

On the basis of the assumption that the response of the vehicle to a sum of uniform waves is equal to the sum of the vehicle responses to the individual components, the frequency of occurrence of certain levels in pitch and heave in ocean-wave environments were calculated.

A vehicle drifting in a fully arisen sea at wind speeds of 10 and 20 knots, representing Hydrographic Office sea states of 2 and 4 without swell, is calculated to have a significant heave of respectively 0.4 and 1.3 feet and a significant pitch of respectively 32 and 77 minutes of arc.

The study indicates that the movements of the vehicle in pitch and heave can be decreased significantly by lowering its flotation chamber and by attaching fins or plates and a weight near its tail end.

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INTRODUCTION

This report covers a theoretical and experimental laboratory study of movements of a large rocket vehicle drifting vertically in wave environments, with a small section of its total height above water. The study was made for the U. S. Naval Missile Center, Point Mugu, California, and executed by the U. S. Naval Civil Engineering Laboratory, Port Hueneme, California, in support of a feasibility study conducted by NMC on the sea launch of large solid-propellant rocket vehicles (Project HYDRA).

The object of this study was to determine the motions of such vehicles in the wave environment of the open sea. The waves in this environment (hereinafter called the "seaway") vary considerably in height, shape, and period dependent on the wind, the water depth, and the wind-affected sea area or fetch.

At present the best representation of the seaway can be made by use of a mathematical model of a random process, which presents no values of the environment as a function of time, but presents statistical values such as averages and the frequency of occurrence of a certain water level. In this approach (Pierson, Neumann, and James¹), the seaway is taken as the sum of a large number of uniform wave trains, each with different amplitudes, and the profiles of the individual wave trains are assumed to be sine curves according to Airy's theory.²

The magnitude of the amplitudes of the wave trains and their distribution over a range of frequencies can be calculated by a standard method from the wind velocity, wind duration, and the fetch. The result of such a calculation can be presented in the form of a wave spectrum, which is a distribution of the mean squares of wave amplitudes of the seaway in a given increment of the frequency over the wave frequencies.

The study of the motions of the vehicle in seaways described by particular spectra was made in two parts. In the first part the responses of the vehicle to uniform waves with different heights and amplitudes were studied theoretically and experimentally. For the experimental work, which was conducted in the NCEL wave tank, NMC furnished two 1-to-120 scale models of an unspecified vehicle (Figures 1 and 2).

In the second part of the study, the responses of the vehicle to seaways, the latter represented by mathematical models of the random process of the ocean surface elevation, were studied, and statistical features of the responses to two particular seaways were obtained.

THEORY

Theory of Movements Induced by Uniform Waves

General. The movements of a rocket vehicle drifting in a wave environment can be described by considering the movements of its mass center relative to a rectangular coordinate system. In analogy to the movements of a ship in a seaway, the movements of the vehicle in its six degrees of freedom are taken as follows (Figure 3):

heave = vertical motion of the mass center
surge = horizontal motion of the mass center in the direction of the wave propagation (drift)
sway = horizontal motion of the mass center normal to the direction of the wave propagation
pitch = angular motion in a vertical plane through the direction of the wave propagation
roll = angular motion in a vertical plane normal to the direction of the wave propagation
yaw = angular motion around the vertical axis

Since the vehicle is symmetrical in any horizontal cross section, it may be assumed that coupling exists only between the wave train and the modes of movement in the vertical plane through the direction of wave propagation; namely, heave, surge, and pitch.

Heave. The heaving motions of the vehicle, which are approximated in the study by a series of cylinders of different radii and length (Figure 4), are analogous with the movements of a dampened mass-spring system, as presented in detail in Appendix A.

The mass in this analogous system (Figure 5) consists of the mass of the vehicle (M) and the so-called added mass (M'') which conveniently may be imagined as an amount of water which is moving together with the vehicle. The damping force of the system is the linearized drag of the vehicle when moving vertically in water without the disturbance of the waves. The restoring force of the system is the restoring force due to buoyancy, which is equal to the weight of the displaced water for any given vertical displacement of the vehicle from its free-floating condition.

In the analogous theory, the excitation consists of three terms (Figure 5), all linear or linearized functions of the wave amplitudes. These three terms have the frequency of the wave, but are not all in phase with it. Consequently, the resultant of these three, which is the excitation of the analogous system, is generally not in phase

with the wave.

The equation of motion of the analogous system is expressed by:

$$(M + M_z'') \ddot{Z} + N_z \dot{Z} + \rho g F_0 Z = F_{ex} \cos(\omega t + \xi + \alpha) \quad (1)$$

(inertia + damping + restoring force = excitation)

where

- M = mass of the vehicle
- M_z'' = added mass of the vehicle in heave
- Z = vertical displacement of the mass center
- \dot{Z} = vertical velocity of the mass center
- \ddot{Z} = vertical acceleration of the mass center
- N_z = damping coefficient in heave
- ρ = density of water
- F_0 = area of cross section at the water line
- F_{ex} = maximum amplitude of the excitation
- $F_{ex} \cos(\omega t + \xi + \alpha)$ = periodic external force on the analogous system (excitation)
- t = time
- ω = frequency of the wave
- ξ = phase angle
- α = phase angle between the wave and excitation

Values of the added mass (M_z'') can be calculated by theory for bodies with a simple geometry, like a sphere or a cylinder (Lamb³, and Wendel⁴). The added mass of more complicated bodies is generally determined from model experiments, or--in the case of ships--calculated from graphs which are based on model experiments. Since the added mass could be obtained from simple model experiments, discussed in Appendix C, no attempt has been made to derive an analytical value.

The damping of the vehicle motions in heave is caused by the generation of waves with the missile center as their origin, and by viscous effects. A theory, deduced in Appendix D, permits the calculation of the damping due to wave generation. In this theory, it is assumed that on the average level of the transitions of the cylinders, which comprise the simplified geometry of the vehicle (Figure 4), periodical sources of fluid exist. These sources are taken in ratio with the movements and the areas. By use of a theory presented by Havelock,⁵ the mean rate of energy outflow by those waves is equaled by the work done by a linear damper, and a value for the damping coefficient due to wave generation is obtained.

The viscous damping is dependent on the geometry of the vehicle and the Reynolds number. Generally, the damping force is a quadric function of the velocity, but Weinblum and St. Denis⁶ indicate that for ship motions the damping may be linearized. This assumption has been made also for the rocket vehicle.

No coefficients were found in the available literature for bodies with a shape similar to the vehicle. By means of a simple test described in Appendix C, a value of the linearized damping coefficients due to viscous and wave-generating effects could be obtained for the vehicle model when moving in its natural frequency.

Since a difference exists between the Reynolds number of the prototype and the Reynolds number of a small scale model, both with comparable movements, responses of the prototype to waves will deviate from the results obtained experimentally with a model. The magnitude of this difference, which is called the scale effect, will be treated in the Discussion.

Besides the amplitude of the heaving motion in waves with particular heights and periods, the movement of the vehicle relative to the water surface is of interest. Namely, the double amplitude of this relative motion indicates the height of the top cylinder, or the last stage of the vehicle, which is alternately exposed to water and air (run-up).

The equation of simple harmonic motion (Equation 1), which resulted from the analogy used, permits the calculations of the phase angle between the excitation and the response of the vehicle. Since the phase angle (α) between the excitation and the wave can be obtained by use of the information presented in Appendix A, the magnitude of the relative motion can be obtained by vectorial representation of displacements and their phase relationship to the wave and by graphical calculation of the resultant of wave and heave displacements.

Pitch and Surge. In the study of the motions of the vehicle (Figures 1 and 2), it appeared convenient also to assume that the vehicle consists of a series of coupled cylinders, as illustrated in Figure 4. After derivations similar to those for heave, presented in Appendix B, it appears that analogous systems can be used for the pitch and surge motions. The motions of these systems, however, are coupled.

The equations of motions, following the form of presentation of the heave, are:

$$\begin{aligned}
(M + M'') \ddot{X} + \sum_{n=0}^n N_n \dot{X} + \sum_{n=0}^n M''_n f_n \ddot{\phi} + \sum_{n=0}^n N_n f_n \dot{\phi} \\
= F' \cos(\omega t + \xi + \alpha')
\end{aligned}
\quad (2)$$

$$\begin{aligned}
\sum_{n=0}^n M''_n f_n \ddot{X} + \sum_{n=0}^n N_n f_n \dot{X} + (I + I'') \ddot{\phi} + \sum_{n=0}^n N_n f_n^2 \dot{\phi} + M g m_p \phi \\
= M'_p \cos(\omega t + \xi + \alpha'')
\end{aligned}
\quad (3)$$

where

- X = horizontal displacement of the mass center
- \dot{X} = horizontal velocity of the mass center
- \ddot{X} = horizontal acceleration of the mass center
- N_n = damping coefficient of a cylinder moving in the X direction
- I = mass inertia moment of the vehicle around its mass center
- I'' = added mass inertia moment
- f_n = distance between the mass center of a cylinder and the mass center of the vehicle
- ϕ = angular displacement of the long axis of the vehicle
- $\dot{\phi}$ = angular velocity of the long axis of the vehicle
- $\ddot{\phi}$ = angular acceleration of the long axis of the vehicle
- m_p = metacenter height
- F' = maximum amplitude of the excitation of the analogous system in surge
- M'_p = maximum amplitude of the excitation of the analogous system in pitch
- n = cylinder number
- M''_n = added mass of a cylinder moving horizontally
- α' = phase angle between wave and excitation in surge
- α'' = phase angle between wave and excitation in pitch

Theory of Movements Induced by a Seaway

In order to describe the movements of the vehicle in a seaway, terms of practical significance to the user have to be selected. At present, descriptions of complex movements are generally given in statistical values of the movement; to conform with this practice, this method of expression has been used in this study.

The calculation method of significant values of responses of systems subjected to random or complex excitations was developed by electrical engineers for use in random noise analyses (Rice,⁷ 1945), and was more recently applied to the calculation of ship movements in a seaway (St. Denis and Pierson,⁸ 1953).

The basic requirement for the applicability of the method is that the response of the considered system is linear to the excitation. If this method is applied to the analogous linear system of the vehicle, as described earlier in connection with Figure 5, then the responses of the vehicle to a sum of uniform wave trains--which are assumed to be moving in one direction--is equal to the sum of the vehicle responses of the individual components.

Since the responses of the vehicle in uniform wave trains can be determined by theory or by model experiments, the complex movements excited by waves with a certain spectrum can be obtained by means of the following procedure, which is illustrated in Figure 6.

For all frequencies of the considered wave spectrum, the value of the mean square of the wave amplitudes per unit of wave frequency--called spectral density of the waves--is multiplied by the corresponding square of the ratio of response to wave amplitude, and the results are plotted as the spectral density of the response. In mathematical terms, the procedure is generally written as follows:

$$S_r(\omega) = S_w(\omega) \cdot [T(\omega)]^2 \quad (4)$$

where $S_r(\omega)$ = spectral density at frequency ω of the response
 $S_w(\omega)$ = spectral density at frequency ω of the waves
 $[T(\omega)]^2$ = square of the ratio of response to wave amplitude at frequency ω of the wave environment

Once the spectrum of the movement in study has been obtained, an integration of the spectrum results in a single number (E), which represents the energy of the movement. If the spectrum is relatively narrow, the average amplitudes of the movement can be expressed in terms of the energy (E), as shown in Table I.

Table I. Response Amplitude (A^{resp}) and Response Energy (E)
Relations (After Pierson, Neumann,⁹ and James,¹ based
on Longuet-Higgins⁹)

Average Response Amplitude Data

Average amplitude of all responses	$A^{\text{resp}}_{\text{av}} = 0.886 \sqrt{E}$
Average amplitude of the 1/3 highest	$A^{\text{resp}}_{1/3} = 1.416 \sqrt{E}$
Average amplitude of the 1/10 highest	$A^{\text{resp}}_{1/10} = 1.800 \sqrt{E}$

Greatest Response Amplitude Probability Data

Number of cycles (N)	Nine times out of ten, the highest amplitude of the response out of a series of N cycles is between			
20	1.40 \sqrt{E}	and	2.44 \sqrt{E}	
50	1.69 \sqrt{E}	and	2.62 \sqrt{E}	
100	1.88 \sqrt{E}	and	2.75 \sqrt{E}	
200	2.05 \sqrt{E}	and	2.87 \sqrt{E}	
500	2.26 \sqrt{E}	and	3.03 \sqrt{E}	
1000	2.41 \sqrt{E}	and	3.14 \sqrt{E}	

Note: The total deviations of the response from minimum
to maximum are twice the values of the amplitudes
in the table.

TEST FACILITIES AND PROCEDURES

The experimental investigations were performed in a water-wave tank 100 feet long, 4 feet high, and 2 feet wide (Figure 7), during the month of June 1960. The wave generator for the tank is of the moving bulkhead type, similar to the machine described by Coyer¹⁰ (1953). The bulkhead is caused to move by a crank wheel and a connecting rod. A variable hydraulic transmission between the driving motor and the crank wheel permits regulation of the frequency of the waves over a large range. The stroke of the bulkhead can be regulated to generate waves of various heights by adjusting the throw of the arm of the crank wheel. A mechanical linkage near the bulkhead permits the adjustment of the horizontal movement of the lower part of the bulkhead according to the calculated value of the orbital movement of the particular waves used in the experiments.

A gravel beach with a one-to-ten slope was installed at the opposite end of the tank to absorb the generated waves and to reduce the generation of standing waves.

The vehicle models furnished by the USNMC for the experimental investigation were wooden models, 17-1/2 inches long and 1-1/4 inches in diameter (Figures 1 and 2), representing a 1-to-120 scale model of a million-pound solid-propellant rocket vehicle, 175 feet long and 12-1/2 feet in diameter.

The movements of one of the models were sensed by use of two displacement gages (Figures 8 and 9). Each gage consisted of a thin wire soldered to two very thin brass plates, one located vertically, the other horizontally. Strain gages were mounted on these plates. Movements of the ends of the wires (arms) of the displacement gages induced strains in the strain gages, which were amplified by carrier amplifiers (Consolidated Engineering Company, Carrier Amplifier System D) and recorded simultaneously with the output of a resistance-type water-level recorder in a high-speed oscillograph (Miller Model H).

It was considered that the displacement gages did not restrain the model in its orbital movements, since the forces to displace the end of the gages were insignificant (approximately 50 milligrams per inch deviation).

To obtain wave run-up information, two parallel wires were glued to the model near the water line (Figure 1, top model). The two wires, together with the water in the tank, were a part of an electrical circuit. Variations in run-up caused a variation in current in the circuit, which was recorded simultaneously with the output of the displacement gages and the water-level recorder. The electrical

connection between this relative-motion gage and the sides was made by very flexible wires with a thickness of 0.003 inch.

The calibrations of the displacement gages and the relative-motion gage were made simultaneously. The model was hung on a hook of a calibration frame. The displacement of the hook, consequently also of the vehicle, could be measured in the horizontal and vertical direction in one-thousands of a foot. At approximately 25 different positions of the hook, the vertical and horizontal coordinates were measured with verniers on the frame and a record of each position was made with the oscillograph. The calibrations were made before and after a series of test runs.

The movements of the model were recorded from the moment the first wave reached the model until obvious irregularities due to standing waves were generated in the tank. Motions were recorded for waves over the range of frequencies (4 through 12 rad/sec) which could be generated in the tank, and for wave heights ranging between 0.3 and 0.7 inches. Higher waves tended to induce vibrations in the displacement gages.

To obtain information about the characteristics of the vehicle in free heave and in free pitch, the model was displaced vertically and horizontally and suddenly released after each displacement. The movements were recorded until they had decayed to insignificant levels.

To check on the results obtained with the displacement gages, photographic observations were made with a second model. This model was equipped with two small light bulbs at the head and the tail (Figure 1, bottom model). Power to these bulbs was supplied by means of two wires with a diameter of 0.005 inch. (These wires were considered not to restrain the motions of the model.) By means of a mechanically operated switch, the light bulb flashed on every 0.15 second during 0.02 second. The movements of the model were photographed at two exposures: one second and one-half second. By means of simple electrical circuits, the flashes and the time of exposure of the photographic film in the camera were recorded on the oscillograph simultaneously with the output of a water-level recorder which was located near the model.

Photographs of the movement of the model were obtained for waves slightly higher and of the same frequency range as those used in the mechanical measurement studies. The scale was obtained by use of white wire grids placed at both sides of the tank (Figure 10). The model was always located one-third of the tank width from the side of the tank nearest the camera.

RESULTS

Still Water - Experimental Results

From the record of the displacement gages over a period of time after the model was displaced vertically in still water and then suddenly released, it appeared that the period of free heave was 2.21 seconds ($\omega = 2.86$ rad/sec). After each cycle of the heaving motion, the amplitude was reduced to 75 percent of the preceding amplitude.

From the record of the displacement gages over a period of time after the model was displaced horizontally and then suddenly released, it appeared that the period of free pitch was 3.0 seconds. After each cycle of the pitching motion, the amplitude was reduced to 78 percent of the preceding amplitude.

Uniform Wave Trains

Experimental Results. Figure 11 shows a typical record obtained with the oscillograph from a test run with the displacement gages. The wave gage was placed 3.18 feet from the model toward the wave generator, consequently the wave record precedes the record of the displacement for the corresponding movements.

Data taken from similar oscillograms are presented in Table II. The wave height and the vehicle responses are obtained as the average of four or five cycles.

Figure 12 presents the results of a photographic observation. The shutter of the camera was opened for approximately one second. The time of the opening was recorded by the oscillograph, together with the time of the light flashes on the head and tail of the model. The output of the wave recorder was also recorded. The position of the head and tail of the model can be identified in the photograph, which is reproduced in Figure 12. In order to show the positions of the vehicle in more detail, the head and tail portions are enlarged ten times, and lines are drawn representing the movements of the long axis enlarged ten times. Data taken from this and similar photographs and corresponding oscillograms are presented in Table III.

Figures 13, 14, and 15 present all experimental results as functions of the wave frequency. (Theoretical results, which will be treated later in this report, are plotted for comparison.) Figure 13 shows the results of all data analyses for heave. The data is presented as plots of the ratios of double heave amplitude to wave height (H_h/H), which equals the ratio of heave amplitude to wave amplitude (A_h/A) as a function of the wave frequency and period. It will be noted that the ratios increase with a decrease in frequency.

Table II. Result of Test With Displacement Gages

Record number	Wave period in seconds (T)	Circular frequency in rad/sec (ω)	Number of waves in test run	Wave height (average) in inches (H)	Heave (double amplitude) in inches (H_h)	Ratio H_h/H	Average deviation from the vertical in minutes of arc	Maximum deviation from the vertical in min/0.1 in wave height	Run-up in inches (H_w)	Ratio H_w/H
1454	0.78	8.0	4	0.62	0.137	0.22	-*	-*	0.42	0.78
1456	0.97	6.5	5	0.53	0.161	0.30	-*	-*	0.38	0.71
1462	0.51	12.4	5	0.31	0.030	0.10	51	8.4	0.25	0.77
1473	0.65	9.7	5	0.40	0.071	0.18	-*	-*	0.37	0.93
1479	0.82	7.7	5	0.41	0.130	0.32	81	9.8	0.35	0.84
1477	0.72	8.7	5	0.39	0.113	0.29	81	10.4	0.35	0.89
1483	1.02	6.2	5	0.38	0.167	0.44	88	11.5	0.32	0.69
1485	1.26	5.0	5	0.30	0.157	0.53	82	13.8	0.23	0.79
1486	1.51	4.2	5	0.23	0.103	0.45	80	17.5	0.15	0.65
1488	1.57	4.0	5	0.36	0.188	0.53	126	17.6	0.27	0.75
1490	1.28	5.0	4	0.47	0.258	0.50	-*	-*	0.32	0.69

*Excessive vibration in the displacement gages

Table III. Result of Photographic Observations

Record number	Wave period in seconds (T)	Circular frequency in rad/sec	Wave height in inches (H)	Heave (double amplitude) in inches (H_h)	Ratio H_h/H	Maximum deviation from the vertical in min. of arc	Maximum deviation from the vertical in min/0.1 in. wave height
1673	0.83	7.6	0.67	0.19	0.29	131	19.6
1674	1.07	5.9	0.655	0.23	0.35	134	20.4
1675	1.08	5.8	0.60	0.26	0.43	120	20.0
1676	1.35	4.6	0.50	0.185	0.37	92	18.4
1782	1.34	4.7	0.48	0.20	0.42	90	18.7
1785	1.06	5.9	0.625	0.19	0.30	122	19.5
1786	0.90	5.3	0.46	0.165	0.36	108	23.4

Figure 14 shows the results of the data analyses of the relative motion of the vehicle and the water surface. The data are presented as plots of the ratio of the run-up, or alternate wet and dry height, to the wave height as a function of the wave frequency and period.

Figure 15 shows the results of all data analyses of the maximum angular deviations from the vertical. The data are presented as plots of the ratio of the maximum angular deviation in minutes of arc to the wave height in tenths of an inch as a function of the wave frequency. The corresponding prototype frequencies and periods are indicated at the top of the graph. The prototype ratios are expressed as the maximum angular deviation in minutes of arc to the wave height per foot.

No measurements were made of sway, yaw, and roll. Visual observations of the movements of the vehicle in uniform waves in the wave tank indicated that these motions were negligible.

In the latter part of the study, movements of the models with modified flotation chambers were visually observed. The flotation chambers were placed at the tail end of the model. To obtain sufficient metacenter height for the vertical stability, a weight was connected rigidly at a certain distance under the chamber (Figure 16). The movements of these models were significantly less than those with the flotation chamber supplied by USNMC (Figure 2). By shifting the weight outside the long axis of the vehicle models, the models could be made to drift with the long axis at a certain angle to the vertical.

Within the range of the wave heights (0.3 to 0.7 inch) used in the experimental investigation, the run-up and the responses of the model in heave and pitch were significantly linear with the wave height. For example, it is estimated that the experimentally obtained ratio of heave to wave height for a wave height of 0.6 inch is approximately 10 percent smaller than the ratio for a wave height of 0.3 inch.

Theoretical Results. Calculations for heave and run-up were made by use of the added mass and damping coefficients obtained in Appendix C from the experimental study of the model motions in still water. The heaving motions of the model represented by Equation 1 and by Equation 17 of Appendix A were calculated and presented in Figure 13, in addition to the experimental results, as values of the ratio of heave amplitude to wave amplitude as a function of the wave frequency. A calculation was also made neglecting the influence of damping. The experimental and theoretical results presented in Figure 13 pertain to vehicle model movements in a water depth of 2.68 feet, which represents a deepwater wave condition for waves with

circular frequencies above 6 radians per second.

The results of an identical calculation for the prototype vehicle movement in deep water, which is water with a depth greater than twice the wave length, is presented in Figure 17. Figure 14 presents, in addition to the experimental data, the results of the theoretical study of the wave run-up along the head of the vehicle model in the wave tank. Plotted is the ratio of the run-up (wetted height) to wave height as a function of the wave frequency and period. Figure 18 presents the results of the theoretical study of the wave run-up on the prototype in deep water. Again, the results are plotted as the ratio of run-up to wave height versus wave frequency and period.

The results of the theoretical study of the heave and the run-up are in good agreement with the results of the tests made in the wave tank. The maximum deviation of the theoretical values from the average values obtained by experiments is approximately 20 percent.

No calculations were made of the pitch and surge of the model. Such calculations can be made by the material presented in Appendix B and by deriving the proper parameters for the mass and damping terms in Equation 2 and in Equation 31 of Appendix B.

Seaway - Theoretical Results

Results of calculations for heave, run-up, and pitch of the prototype vehicle acted upon by nonuniform waves in deep water are presented in Table IV. A fully arisen sea induced by wind speeds of 10 and 20 knots was selected as a typical seaway. These wave environments represent conditions for sea states (Hydrographic Office Scale) 2 and 4, without swells.

The calculations were made by use of the results of the theoretical study of heave and run-up of the vehicle in uniform deepwater waves (Figures 17 and 18) and the results of the experimental study for pitch (Figure 15). Figures 19, 20, and 21 present the calculations according to the pattern presented in Figure 6 for the vehicle movements in heave, pitch, and run-up in a fully arisen sea at a wind speed of 20 knots.

Table IV. Results of Calculations for Heave, Run-up, and Pitch in a Fully Arisen Sea

Characteristic Statistical Value	Fully arisen sea with wind speed of 20 knots (sea state 4)			Fully arisen sea with wind speed of 10 knots (sea state 2)		
	Heave (double amplitude) in ft	Run-up (total) in ft	Pitch (maximum deviation from the vertical) in min. of arc	Heave (double amplitude) in ft	Run-up (total) in ft	Pitch (maximum deviation from the vertical) in min. of arc
Average	0.8	4.3	47	0.3	2.2	20
Average 1/3 highest (significant)	1.3	6.8	77	0.4	3.4	32
Average 1/10 highest	1.7	8.7	97	0.5	4.4	40
In series of 1000 cycles*, 9 times out of 10 the highest value is between	2.2 and 2.9	10.6 and 15.1	130 and 168	0.7 and 0.9	5.9 and 7.6	53 and 71

*For a fully arisen sea with a wind speed of 20 or 10 knots, the respective time for 1000 cycles is approximately 2-1/4 or 1-1/2 hours.

DISCUSSION

Heave

It appears from Figure 13 that the model movements in the wave tank can be calculated by means of Equation 1, with an error of less than 20 percent for frequencies larger than 5 radians per second. The discrepancy of theoretical and experimental results may be explained by the approximations made in the derivation of the equation of motion. In particular, the calculation of the mean average velocity and acceleration of the water mass displaced by the vehicle in the excitation (Equations 7 and 8, Appendix A) is considered rather crude by the author, and it is suggested that a more refined method, as outlined in Appendix A be used in future investigations. (Equation 11)

A considerable approximation has also been made initially by following the calculation method for ship motions in uniform waves⁶ and linearizing the damping term in Equation 1 (Weinblum and St. Denis,⁶). In ship movements, as indicated by Havelock,¹¹ Korvin-Kroukovsky,^{12, 13} and Gerritsma,¹⁴ the damping is mainly contributed by the more or less linear effect of wave generation by the ship. However, the results of the theoretical investigation of the damping of the vehicle model due to wave generation, presented in Appendix D, indicate that the damping due to this effect is small and that total damping is mainly due to viscous effects.

Consequently, because of differences in the frictional effects of the prototype and model, considerable difficulties should be expected in scaling the model towards the prototype characteristics by use of Froude's Law. Fortunately, consideration of the derived analogous system indicates that damping is of minor importance over a large range of frequencies.

It will be noted that Equation 1 represents the classical equation for the forced motion of a mass-spring system. The amplitudes of the motion of such a system are not significantly influenced by values of the damping term for harmonic movements with frequencies larger than twice the natural frequency. A numerical calculation of the excitation of the analogous system (right side of Equations 1 and 17, which also contain a term for the viscous effect, or drag) indicates that its values are of minor importance for frequencies larger than twice the natural frequency in heave. The insignificance of the combined effect of the damping term and the drag term for frequencies larger than twice the natural frequency is illustrated in Figure 13, which presents the results of the heave amplitude investigation for the model, calculated with the experimentally obtained damping term and without any damping. Consequently, the results of the model

study for circular frequencies larger than 5 radians per second are representative for the prototype for the corresponding frequencies, obtained by use of Froude's Law, larger than 0.46 radians per second (periods smaller than 13 seconds).

It is noted that a difference exists in the theoretical results for the prototype in deep water (Figure 17) and the scaled results of the model investigation in the wave tank for frequencies smaller than 6.2 radians per second (prototype periods larger than 11 seconds). In that case, the particle motions of such waves are effected by the bottom, the orbits of those particles are elliptical rather than circular, and consequently a different movement of the vehicle is induced.

Waves with frequencies close to the frequency of the free heave may induce excessive movements whose magnitude depends on the damping. Since the frequency of free heave is low for large vehicles, swells may induce the major heaving motion. The large heave (approximately 5 feet) with long periods (approximately 16 seconds) of a long and slender vehicle (HYDRA II) drifting in a vertical attitude prior to launching, observed by the author off Point Mugu, California, in an environment with waves with an estimated significant height of 1 to 1-1/2 feet and periods of approximately 5 seconds, were likely induced by swells with a period close to the period of the free heave of the vehicle and with a very small height which could not be visually determined.

The movements in the frequency close to the frequency of free heave can be decreased by increasing the damping. This can be accomplished by installing horizontal plates or fins on the outer surface of the vehicle, preferably close to the tail end. Besides an increase in damping, such plates will increase the added mass, and consequently a lower natural frequency in heave will be obtained.

Since the results of the movement study in uniform waves are limited to waves with periods shorter than 13 seconds, spectral calculations, such as presented in Figure 19, must be limited to seaways which contain no waves with significant amplitudes which have periods longer than 13 seconds.

Run-up

The ratio of wave run-up to wave height at a particular frequency for uniform waves depends on the ratio of heave to wave height and the phase relation between wave and heave. As discussed under Heave, the first of these ratios is practically independent of values of the damping terms in Equations 1 and 17.

A study of the vectorial representation of excitation and heaving movements of the analogous system indicates that the magnitude of the phase angle between heave and wave, which depends on the damping characteristics, does not significantly influence the ratio of run-up to wave height for frequencies larger than 0.46 radians per second (periods shorter than 13 seconds). Consequently, the theoretical results of the vehicle moving in uniform waves in deep water may be applied to spectral calculations of the run-up in seaways such as presented in Figure 18, if the spectrum of the seaway does not contain waves of significant height with periods longer than 13 seconds.

Pitch

It will be noted in Equations 2 and 3 that the vehicle movements are analogous to the movements of a torsional mass-spring system, excited by a forced oscillation and coupled to another mode of movement of the vehicle (surge).

The restoring force of this system is very small because of the very small cross section of the vehicle at the water line. This force may be neglected for frequencies higher than twice the natural frequency of the system, in which case Equations 2 and 3 would contain only acceleration and velocity terms.

The analyses of the records and photographs of the movement (for example, Figure 12) indicated that the phase angle between pitch and surge, respectively, and the horizontal particle motion is very small and can be assumed to be zero. On the basis of the assumptions that, by scaling according to Froude's Law, the N values of the cylinders relative to each other do not change significantly, a vectorial representation of Equation 2 and Equation 3 indicates that the scale effect will be small. Consequently, the angular movements in pitch can be considered representative for the prototype.

Nevertheless, the author is of the opinion that a theoretical study should be made of the movements in pitch and surge according to the outline given in Appendix B.

Under the discussion of heave it is indicated that, for frequencies higher than 6.2 radians per second, the particle motions in the wave tank are effected by the bottom and that those particles describe elliptical rather than circular orbits. Numerical calculations indicate that the horizontal displacement for frequencies higher than 0.46 radian per second are essentially the same as for deepwater waves. Consequently, for frequencies higher than 0.46 radians per second, the results of the model study are representative

for the prototype in deep water after scaling by use of Froude's Law, and those results may be applied to spectral calculations, if the spectrum of the waves contains no waves of significant height with periods longer than 13 seconds.

In the spectral calculations of pitch (for example, as presented in Figure 20), it is assumed that all waves are propagating in one direction. In reality, however, waves are generated from different directions, but only a very small part of the wave energy is in waves which make a significant angle (larger than approximately 15 degrees) from the main direction (Marks¹⁵). It is possible to determine the movements of the vehicle in such waves by calculating the responses at the different directions of the spectrum and by a vectorial addition of the components thus obtained. Since the spread of the main wave directions is relatively small, the spread of the responses is small and its effect can be disregarded.

Reduction of Movements

Since the particle motions near the surface are larger than those at a greater depth, a reduction of the dimensions of the vehicle near the surface will tend to reduce the movements in pitch.

An improvement can be obtained by relocating the flotation chamber near the tail end of the vehicle (Figure 16). Since this would change the location of the metacenter height unfavorably, a correction would have to be made by installing a weight near the bottom of this chamber. If this weight is made in the shape of a plate, considerable damping would be obtained in the heaving motion. This would tend to reduce the free heave amplitude and frequency. The visual observation made in the wave tank with such a vehicle confirmed this analysis.

If a reduction of the movements of the vehicle is required, a further experimental and theoretical study of the shape, size, and location of the flotation chamber would be desirable.

SUMMARY OF FINDINGS

1. From the observations in a wave tank of the motions of a 1-to-120 scale model of a rocket vehicle drifting in uniform wave trains with heights ranging from 0.3 to 0.7 inch and with periods ranging from 0.5 to 1.6 seconds, it is found that:

- a. The heave, run-up, and pitch are functions of the wave period.

- b. The heave, run-up, and pitch are linear functions of the wave height.
 - c. The ratio of heave amplitude to wave amplitude is 0.1 for waves with a period of 0.55 second, and 0.4 for waves with a period of 1.2 seconds (Figure 13).
 - d. The ratio of run-up to wave height is 0.9 for waves with a period of 0.55 second, and 0.7 for waves with a period of 1.2 seconds (Figure 14).
 - e. The maximum deviation from the vertical is 9 minutes of arc for waves with a height of 0.1 inch and a period of 0.55 seconds, and 11 minutes for waves with a height of 0.1 inch and a period of 1.2 seconds (Figure 15).
2. The results of the theoretical study of the heave and run-up are in good agreement with the results of the experimental study in the wave tank.
3. Froude's scaling law is applicable for the heave, run-up, and pitch of the model in waves with periods shorter than 1.2 seconds.
4. The vehicle drifting in uniform waves in deep water has:
- a. A ratio of heave amplitude to wave amplitude of 0.1 for waves with a period of 6 seconds, and 0.4 for waves with a period of 12 seconds (Figure 17).
 - b. A ratio of run-up to wave height of 0.9 for waves with a period of 6 seconds, and 0.6 for waves with a period of 12 seconds (Figure 18).
 - c. A ratio of maximum angular deviation to wave height of 9 minutes of arc per foot for waves with a period of 6 seconds, and 11 minutes of arc per foot for waves with a period of 12 seconds (Figure 14).
5. The vehicle drifting in a fully arisen sea caused by wind speeds of 10 and 20 knots--representing sea states 2 and 4 without swell--has, on the basis of the assumption that the response of the vehicle to a sum of uniform waves is equal to the sum of the vehicle responses to the individual components (Table IV):
- a. An average heave of respectively 0.3 and 0.8 foot; a significant heave of respectively 0.4 and 1.3 feet; a chance of nine out of ten that the highest heave

during an interval of 1-1/2 and 2-1/4 hours is respectively between 0.7 and 2.2 feet, and 0.9 and 2.9 feet.

- b. An average run-up of respectively 2.2 and 4.3 feet; a significant run-up of respectively 3.4 and 6.8 feet; a chance of nine out of ten that the highest run-up during an interval of 1-1/2 and 2-1/4 hours is respectively between 5.9 and 10.6 feet, and 7.6 and 15.1 feet.
 - c. An average maximum angular deviation from the vertical (pitch) of respectively 20 and 47 minutes of arc; a significant pitch of respectively 32 and 77 minutes; a chance of nine out of ten that the maximum pitch during an interval of 1-1/2 and 2-1/4 hours is respectively between 53 and 71 minutes, and 130 and 168 minutes.
6. The movements in pitch of the vehicle are reduced significantly by installing a flotation chamber and a weight at the tail end of the vehicle; the movements in heave at frequencies close to the frequency of free heave are reduced by installing a damping plate near the tail end.

CONCLUSIONS

1. For the vehicle drifting in a vertical attitude in a seaway, the occurrence of certain levels of movement in heave and pitch and the run-up along the head can be calculated by use of Figures 17, 14, and 18 on the basis of the assumption that the response of the vehicle to a sum of uniform waves is equal to the sum of the vehicle responses to the individual components, if the spectrum of the seaway contains no waves of significant amplitudes with period longer than 13 seconds.
2. The movements of the vehicle in pitch can be reduced by installing a flotation chamber and a weight at the tail of the vehicle; the movements in heave at frequencies close to the frequency of free heave can be reduced by installing a damping plate near the tail end.

SUGGESTIONS FOR FUTURE RESEARCH

1. That a theoretical study be made of the movements of the vehicle in pitch and surge in order to obtain information about these movements for waves with periods longer than 13 seconds, and to supplement the experimental results.
2. That displacement histories of free heave and free pitch of prototypes or full-scale models be measured, from which information about damping and drag characteristics can be obtained for use in the theoretical study.
3. If a reduction of the movements of the vehicle is required, that theoretical and experimental laboratory studies be made of vehicles with the flotation chamber near the tail end.

Appendix A

THEORY OF THE HEAVING MOTION

The shape of the vehicle is approximated by a series of cylinders of different dimensions, as shown in Figure 2, and is drifting in a uniform wave train with waves of low height.

The origin of the coordinate system that will be used to describe the waves according to Airy's theory (Wiegel and Johnson¹⁶) is taken at the still-water level in the axis of the series of cylinders. The X axis is taken in the direction of the wave propagation, the Z axis vertically.

The waves can be described by:

$$\eta = A \cos(kx - \omega t - f) \quad (5)$$

where A = amplitude of the wave, which is equal to half the wave height

$$k = 2\pi/\lambda$$

λ = wave length

$$\omega = \text{frequency } (= 2\pi/T)$$

f = phase angle

η = elevation

Referring to Weinblum and St. Denis⁶, if Z is the upward displacement of the mass center of the vehicle, the equation of motion can be written:

$$F_z(p) = M \ddot{Z} + M_z'' (\ddot{Z} - \dot{W}) - \frac{C_d}{2} \rho F (W - \dot{Z})^2 + M g \quad (6)$$

where $F_z(p)$ = total vertical upward force from the wave pressures on the horizontal planes of the vehicle

M = mass of the vehicle

M_z'' = added mass in heaving motion

C_d = drag coefficient

F = significant cross section of the vehicle

ρ = density of water

g = acceleration due to gravity

W = mean vertical velocity of the water mass displaced by the vehicle

\dot{W} = mean vertical acceleration of the water mass displaced by the vehicle

The added mass (M'') may be visualized in many cases as an amount of water which is moved with the vehicle. The theory of this added mass (Wendel, Lewis,¹) is complicated and a discussion about it is not in the scope of this investigation. The added mass depends on the shape of the object in relation to the fluid flow around it. Consequently, the added mass of the vehicle in vertical motion is significantly different from the added mass in horizontal motion.

Assuming that the radii of the cylinders of the vehicle are small compared to the length of the wave and that the radii of the cylinders are of the same order, the quantities W and \dot{W} may be defined as:

$$W = \frac{1}{D_4} \int_{D_4}^{\eta} w \, dz \quad (7)$$

$$\dot{W} = \frac{1}{D_4} \int_{D_4}^{\eta} \dot{w} \, dz \quad (8)$$

where D_n = distance from the still-water surface to a horizontal plane of the missile (Fig. 4)

$$w = A \omega \frac{\sinh k(z+d)}{\sinh kd} \sin(kx - \omega t - \frac{\pi}{2})$$

$$\dot{w} = A \omega^2 \frac{\sinh k(z+d)}{\sinh kd} \cos(kx - \omega t - \frac{\pi}{2})$$

d = depth

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{2g}{\lambda} \tanh \frac{d}{L}}$$

Evaluation of the integrals in Equations 7 and 8 gives, with some approximation:

$$W = - \frac{A q [\cosh kd - \cosh k(d-D_4)]}{\omega D \cosh kd} \sin(\omega t + \frac{\pi}{2}) \quad (9)$$

$$\dot{W} = - \frac{A q [\cosh kd - \cosh k(d-D_4)]}{D \cosh kd} \cos(\omega t + \frac{\pi}{2}) \quad (10)$$

Equations 9 and 10 were used in the calculation of the heaving motion of the vehicle. More accurate values may be obtained by integration over the individual cylinders. For example, the average velocity can be expressed by:

$$W = \frac{\rho F_0}{M} \int_{D_1}^{\eta} w dz + \frac{\rho (F_0 + F_1)}{M} \int_{D_2}^{D_1} w dz + \frac{\rho (F_0 + F_1 - F_2)}{M} \int_{D_3}^{D_2} w dz + \frac{F_4}{M} \int_{D_3}^{D_4} w dz \quad (11)$$

The pressure at a depth z is given by:

$$p = p_a - \rho g z + \rho g \eta \frac{\cosh k(z+d)}{\cosh kd} \quad (12)$$

where p_a = atmospheric pressure

Assuming that the pressures on each area of the horizontal planes of the vehicle are equal to those of the center of that plane, the force $F_z(p)$ is found by algebraic summation of pressures on the four horizontal planes (Figure 5):

$$F_z(p) = - (p_{z_1} - p_a) F_1 + (p_{z_2} - p_a) F_2 - (p_{z_3} - p_a) F_3 + (p_{z_4} - p_a) F_4 \quad (13)$$

where $z = Z - D$

$$F_z(p) = M g - \rho g F_0 Z + \frac{\rho g \eta}{\cosh kd} \left\{ \cosh k(Z+d) \left[- F_1 \cosh kD_1 + F_2 \cosh kD_2 - F_3 \cosh kD_3 + F_4 \cosh kD_4 \right] \right\}$$

$$\begin{aligned}
& - \sinh k(Z+d) \left[- F_1 \sinh kD_1 + F_2 \sinh kD_2 - F_3 \sinh kD_3 \right. \\
& \left. + F_4 \sinh kD_4 \right] \} \quad (14)
\end{aligned}$$

Since the displacements are small compared to the depth of the water, the displacements (Z) in the term on the right side may be disregarded, thus:

$$\cosh k(Z+d) = \cosh kd \quad (15)$$

$$\sinh k(Z+d) = \sinh kd \quad (16)$$

If the third term on the right side of Equation 6, which represents the damping, is linearized and by use of Equations 14, 15, and 16:

$$(M + M_z'') \ddot{Z} + N \dot{Z} + \rho g F_0 Z = M'' \dot{W} + N W + \frac{\rho g \eta}{\cosh kd} (Q) \quad (17)$$

$$\begin{aligned}
\text{where } Q = & \cosh kd \left[- F_1 \cosh kD_1 + F_2 \cosh kD_2 - F_3 \cosh kD_3 \right. \\
& \left. + F_4 \cosh kD_4 \right] - \sinh kd \left[- F_1 \sinh kD_1 + F_2 \sinh kD_2 \right. \\
& \left. - F_3 \sinh kD_3 + F_4 \sinh kD_4 \right]
\end{aligned}$$

Equation 15 represents the movements of a dampened linear mass-spring system with the periodical excitations (Figure 5):

$$F_{ex}^h = M'' \dot{W} + N W + \frac{\rho g \eta}{\cosh kd} (Q) \quad (18)$$

It will be noted in Equation 17 that the last term on the right side represents the excitation by the pressure variation of the wave on the horizontal planes of the vehicle. The second term represents the excitation by the flow of the water around the body in the vertical direction (drag). The first term on the right side is difficult to visualize. In essence, it represents an inertia excitation, which is the added mass times the acceleration of the water particles displaced by the mass of the vehicle.

All three terms are periodic forces with the frequency of the wave. It will be noted from Equations 9, 10, and 17 that these periodic forces are not all in phase with the wave. The magnitude of the total periodic force and its phase angle relative to the wave can be calculated by a vectorial summation of the three components of the excitation. Consequently, the excitation may be expressed as:

$$F_{ex}^h = F_{ex} \cos(\omega t + \phi) \quad (19)$$

Appendix B

THEORY OF THE PITCHING AND SURGING MOTIONS

In the following analyses it is assumed that the vehicle consists of a series of coupled cylinders, each with different length and diameter. Next, it is assumed that the horizontal displacement of each cross section of a cylinder is equal to the displacement of the center.

Using the same coordinate system as is used for the analysis of the heaving motion, the equations of the surge (X) and pitch (ϕ) are expressed by:

$$\sum_{n=0}^{n=n} F_n(p) = M \ddot{X} + \sum_{n=0}^{n=n} M_n (\dot{U}_n - \ddot{X} - f_n \ddot{\phi}) + \frac{1}{2} \sum_{n=0}^{n=n} C_d \rho L_n^2 R_n (U_n - \dot{X} - f_n \dot{\phi})^2 \quad (20)$$

$$\sum_{n=0}^{n=n} F_n(p) f_n = I \ddot{\phi} + \sum_{n=0}^{n=n} M_n f_n (\dot{U}_n - \ddot{X} - f_n \ddot{\phi}) + \frac{1}{2} \sum_{n=0}^{n=n} C_d \rho L_n^2 R_n f_n (U_n - \dot{X} - f_n \dot{\phi})^2 \quad (21)$$

where $F_n(p)$ = total horizontal force from the wave pressure on a cylinder
 M_n = mass of a cylinder moving horizontally
 L_n = length of a cylinder
 R_n = radius of a cylinder
 U = mean horizontal velocity of the water mass displaced by a cylinder
 \dot{U} = mean horizontal acceleration of the water mass displaced by a cylinder

On the basis that the wave amplitudes are relatively small, the horizontal velocity (u) and the horizontal acceleration (\dot{u}) of the water particles are, respectively:

$$u = A \omega \frac{\cosh k(z+d)}{\sinh kd} \cos(kx - \omega t - \xi) \quad (\text{Wilson}^{18}) \quad (22)$$

$$\dot{u} = A \omega^2 \frac{\cosh k(z+d)}{\sinh kd} \sin(kx - \omega t - \xi) \quad (23)$$

Accordingly, the mean velocity (U_n) and the mean acceleration (\dot{U}_n) are defined:

$$U_n = 2 \int_0^\pi \frac{1}{2 R_n (D_n - D_{n-1}) \cos \gamma} \int_{x=-R \cos \gamma}^{x=R \cos \gamma} \int_{z=D_n}^{z=D_{n-1}} u \, dx \, dz \, d\gamma \quad (24)$$

$$\dot{U}_n = 2 \int_0^\pi \frac{1}{2 R_n (D_n - D_{n-1}) \cos \gamma} \int_{x=-R \cos \gamma}^{x=R \cos \gamma} \int_{z=D_n}^{z=D_{n-1}} \dot{u} \, dx \, dz \, d\gamma \quad (25)$$

where γ = angle in the horizontal plane

For one cylinder, the total wave pressure may be written:

$$F_n(p) = 2 \int_0^{\pi} \left\{ \left[\int_{D_n}^{D_{n-1}} (p - p_a) dz \right]_{x=-R_n \sin \gamma} - \left[\int_{D_n}^{D_{n-1}} (p - p_a) dz \right]_{x=R_n \sin \gamma} \right\} R \gamma \sin \gamma d\gamma \quad (26)$$

where the wave pressure (p) is given by:

$$p = p_a - \rho g z + \rho g \gamma \frac{\cosh(z+d)}{\cosh kd} \quad (27)$$

Evaluation of the integrals in Equation 26 by use of Equation 27 gives:

$$\sum F_n(p) = \sum M' \dot{U}_n \quad (28)$$

$$\sum F_n(p) f_n = \sum M'_n f_n \dot{U}_n - M g m_\phi \phi \quad (29)$$

where m_ϕ = metacenter height.

By use of Equation 28 in Equation 20 and Equation 29 in Equation 21, and by linearizing the damping terms, the equations of motions may be expressed:

$$\begin{aligned} \sum (M_n + M''_n) \ddot{X} + \sum N_n \dot{X} + \sum M''_n f_n \ddot{\phi} + \sum N_n f_n \dot{\phi} = \\ + \sum (M_n + M''_n) \dot{U} + \sum N_n U \end{aligned} \quad (30)$$

$$\begin{aligned}
& (I+I'')\ddot{\phi} + \sum N_n f_n^2 \dot{\phi} + M g m_{\phi} \phi \\
& + \sum M_n'' F_n \ddot{X} + \sum N F_n \dot{X} = \sum (M_n + M_n'') f_n \dot{U}_n + \\
& \sum N_n f_n U_n
\end{aligned} \tag{31}$$

where m_{ϕ} = metacenter height

It will be noted that Equation 30, (pitch equation) represents a dampened mass-spring system. The excitation of this system is coupled with the movement in surge.

Appendix C

CALCULATION OF THE ADDED MASS AND THE DAMPING FACTOR FROM THE FREE-HEAVE TESTS

Since no excitations are present, the equation of motion in free heave may be written:

$$(M+M'') \ddot{Z} + N \dot{Z} + \rho g F_0 Z = 0 \tag{32}$$

Referring to the handbooks on vibration (Den Hartog¹⁹, Hansen and Chenea²⁰, and Timoshenko²¹), the natural frequency in free heave (ω_n) is expressed by:

$$\omega_n = \sqrt{\frac{\rho g F_0}{(M+M'')}} \tag{33}$$

Evaluation of Equation 32 by use of the measured natural frequency ($\omega_n = 2.86$ rad/sec) and the numerical values of F_0 and M presented in Figure 4 gives:

$$M_z'' = 0.15 M \tag{34}$$

The ratio of subsequent amplitudes of the decaying motion of the free-heave oscillation (Hansen and Chenea²⁰) is:

$$\frac{[A_h]_{t+\tau}}{[A_h]_t} = e^{-\frac{\pi N}{(M+M'')}} = e^{-\delta} \quad (35)$$

where $[A_h]_t$ = free-heave amplitude at time t

$[A_h]_{t+\tau}$ = free-heave amplitude at time $t +$

τ = time of one complete cycle of the oscillation

δ = logarithmic decrement

By use of the results of the free-heave test, a numerical evaluation of Equation 35 gives for the model:

$$\delta = 0.25$$

$$N = 0.46 \text{ lb/ft/sec}$$

Appendix D

CALCULATION OF THE DAMPING IN FREE HEAVE DUE TO WAVE GENERATION

The damping of the motion of the vehicle arises partly from fluid frictional effects and partly from energy losses by generation of waves which are propagating in concentric circles with the center of the vehicle as their origin.

The energy radiated by a cylinder of radius (b), immersed with its vertical axis to a depth (d), and making a forced vertical harmonic motion with an amplitude (A) and frequency (ω), is, according to Havelock⁵:

$$E' = \frac{\pi^2 \rho}{4 g} b^4 A^2 \omega^5 e^{-2\omega^2 d/g} \quad (36)$$

where E' = mean rate of transmission of energy.

Equation 36 can be written:

$$E' = \frac{\rho}{4g} F^2 A_f^2 \omega^5 e^{-2\omega^2 d/g} \quad (37)$$

where F = area at the bottom of the cylinder.

If all the horizontal areas at the transition of the vehicle cylinders are considered as sources of radiation, the total mean rate of radiation (E_t) is:

$$E_t = \frac{\rho}{4g} \omega^5 A_f^2 \sum_{n=1}^{n=4} F_n^2 e^{-2\omega^2 d_n/g} \quad (38)$$

where F_n = horizontal area (Figure 4)

d_n = depth of considered area under the still-water level

Following the method of calculation of the N coefficient for the natural damped oscillations as outlined by Havelock, Equation 38 becomes:

$$\frac{1}{2} N_r A_f^2 \omega_n^2 = \frac{\rho}{4g} \omega_n^5 A_f^2 \sum_{n=1}^{n=4} F_n^2 e^{-2\omega_n^2 d_n/g} \quad (39)$$

or

$$N_r = \frac{\rho}{2g} \omega_n^3 \sum_{n=1}^{n=4} F_n^2 e^{-2\omega_n^2 d_n/g} \quad (40)$$

where N_r = damping coefficient due to wave generation.

Evaluation of Equation 40 by use of the measured natural frequency and the model dimensions (Figure 4) gives:

$$N_r = 0.95 \times 10^{-4} \text{ lb/ft/second}$$

A comparison with the value of the total damping obtained in Appendix C from experiments indicates that the damping due to wave generation may be neglected.



Figure 1. Scale models used in the experimental study.

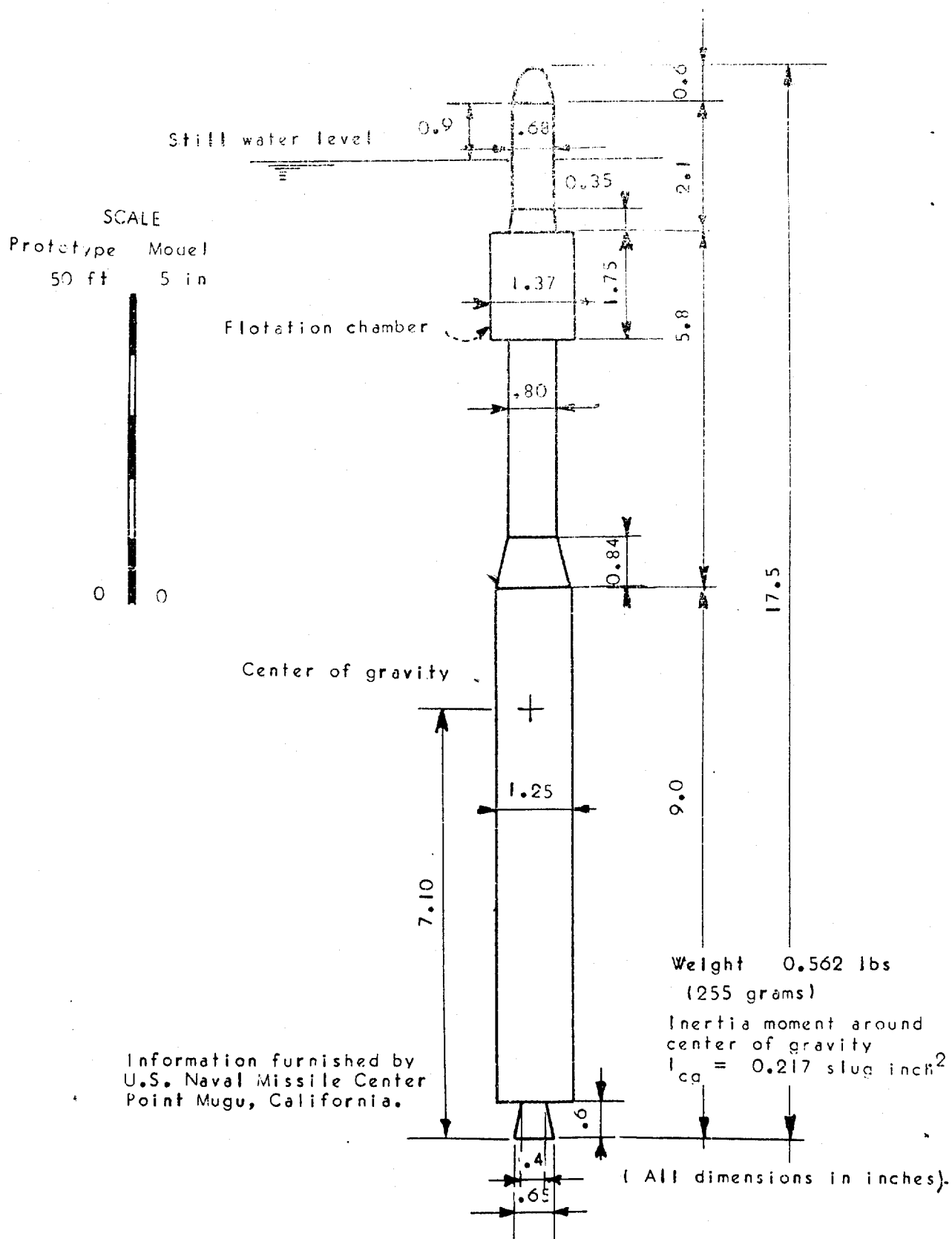


Figure 2. Dimensions of the vehicle model.

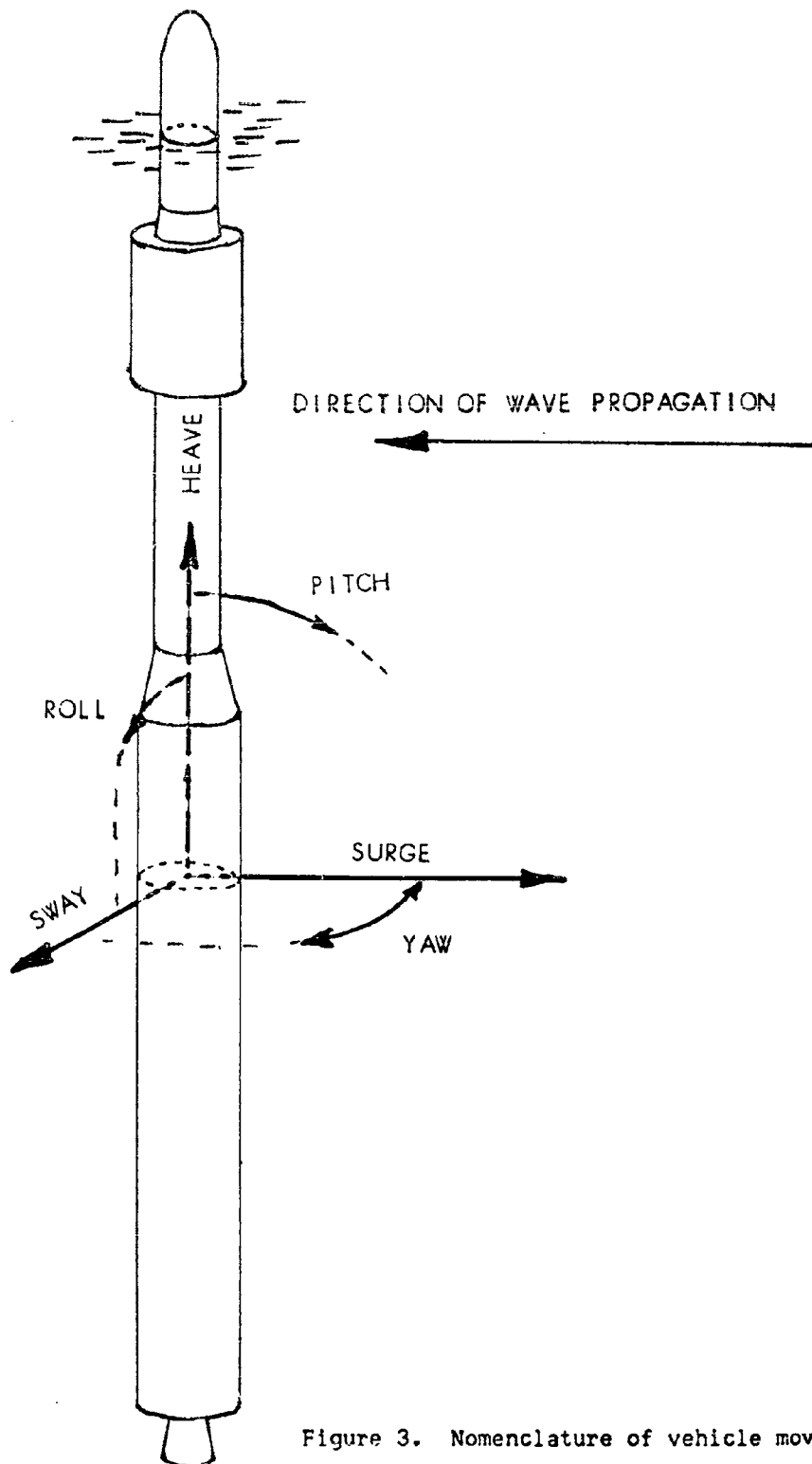


Figure 3. Nomenclature of vehicle movements.

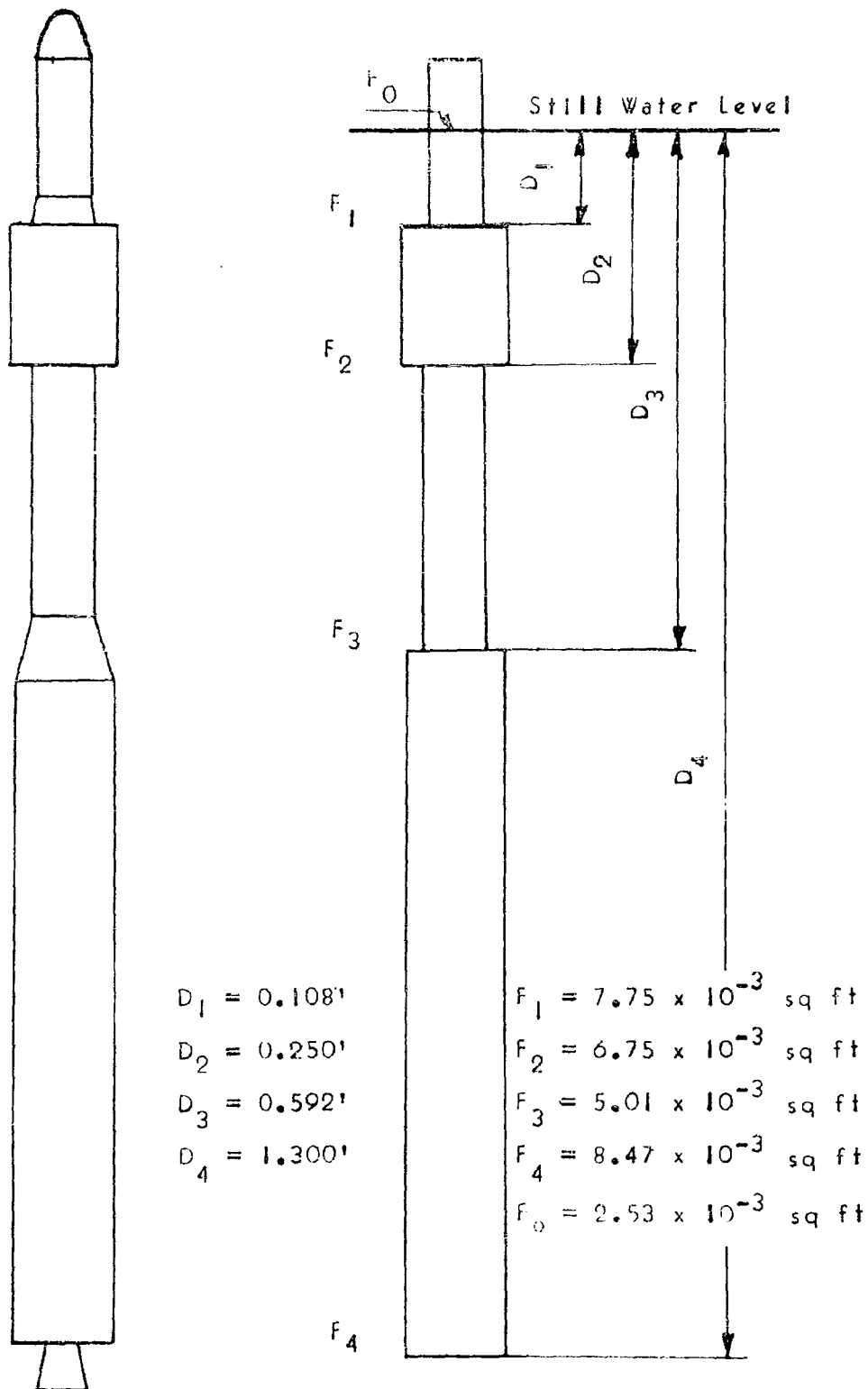
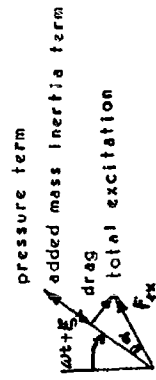
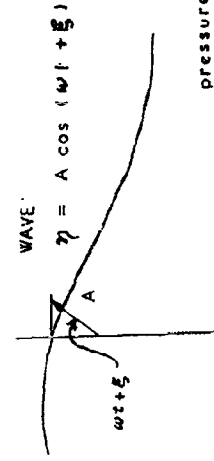
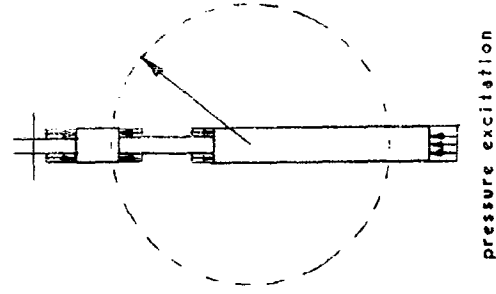
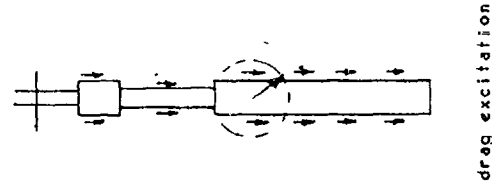
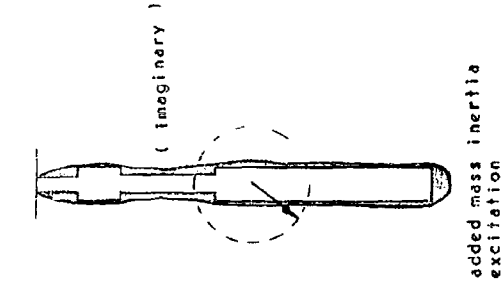
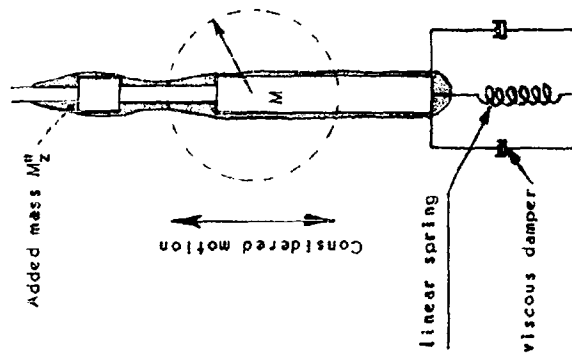


Figure 4. Approximation of the model dimensions for the analysis of the heaving motion.

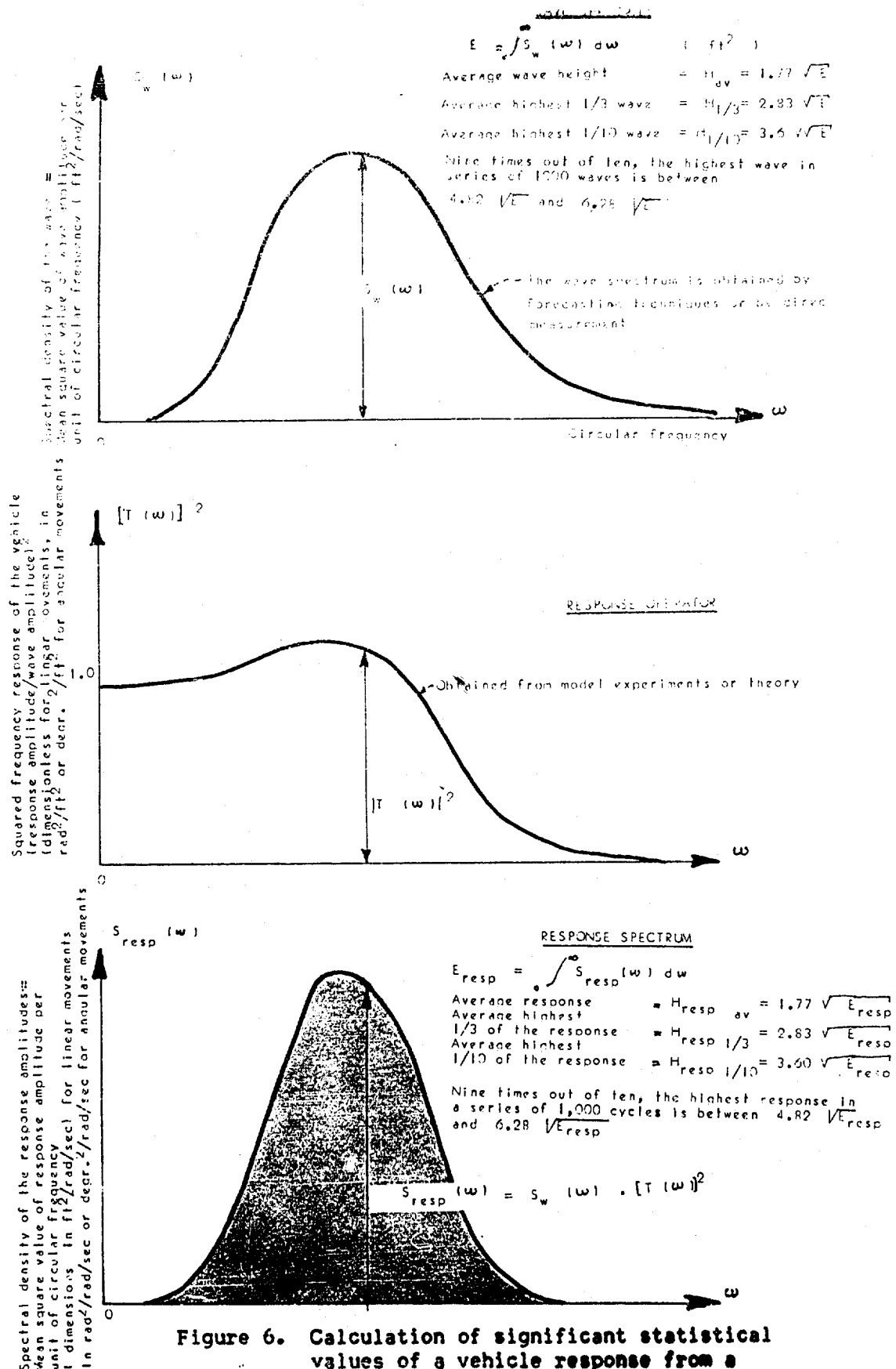


vectorial summation of excitations

$$(M + M_Z^a) \ddot{Z} + N \dot{Z} + \rho g F_0 Z = F_{ex} \cos(\omega t + \xi + \alpha)$$

(inertia + damping + restoring) force = excitation

Figure 5. Mechanical analogy of the vehicle in heaving motion.



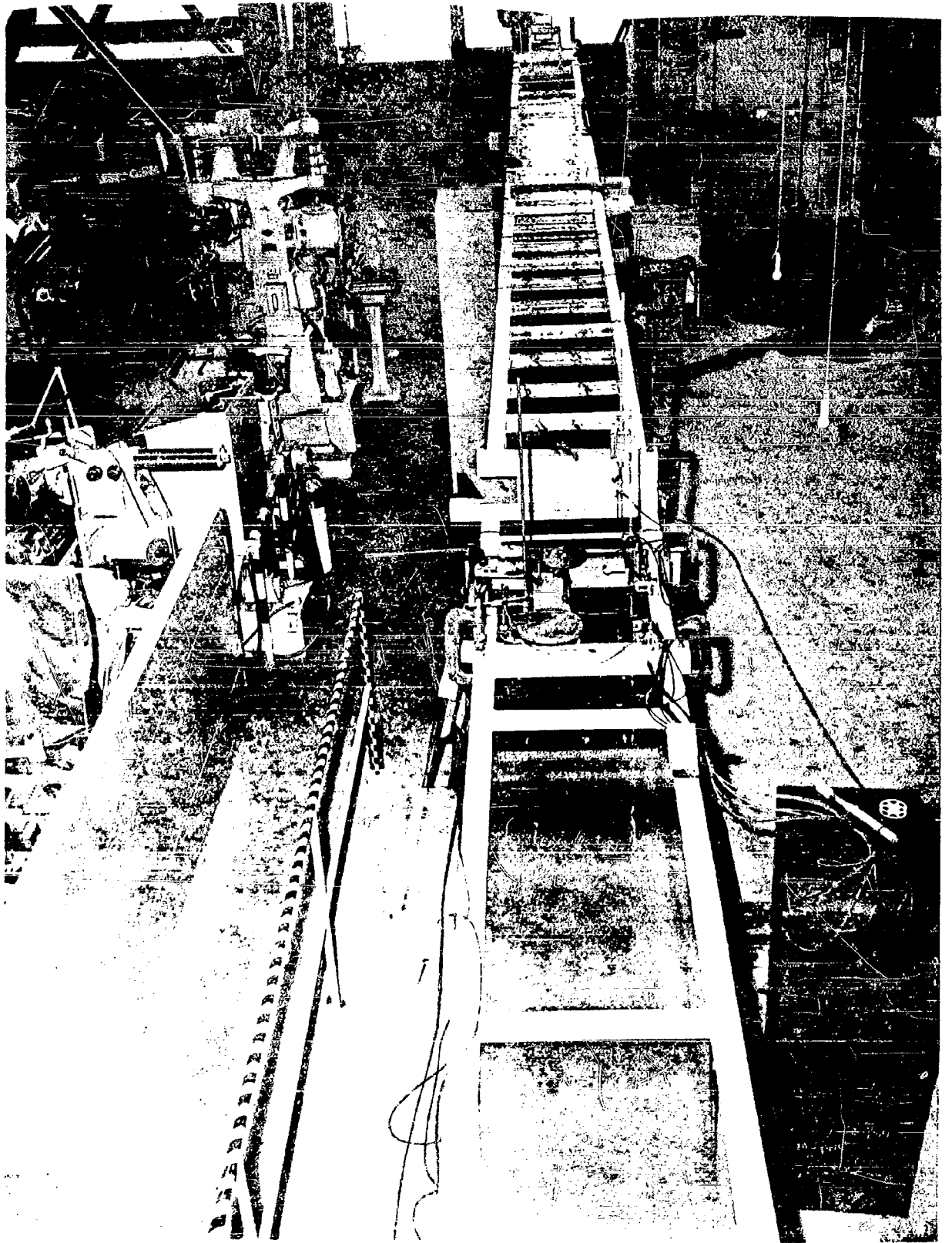


Figure 7. NCEL wave tank.

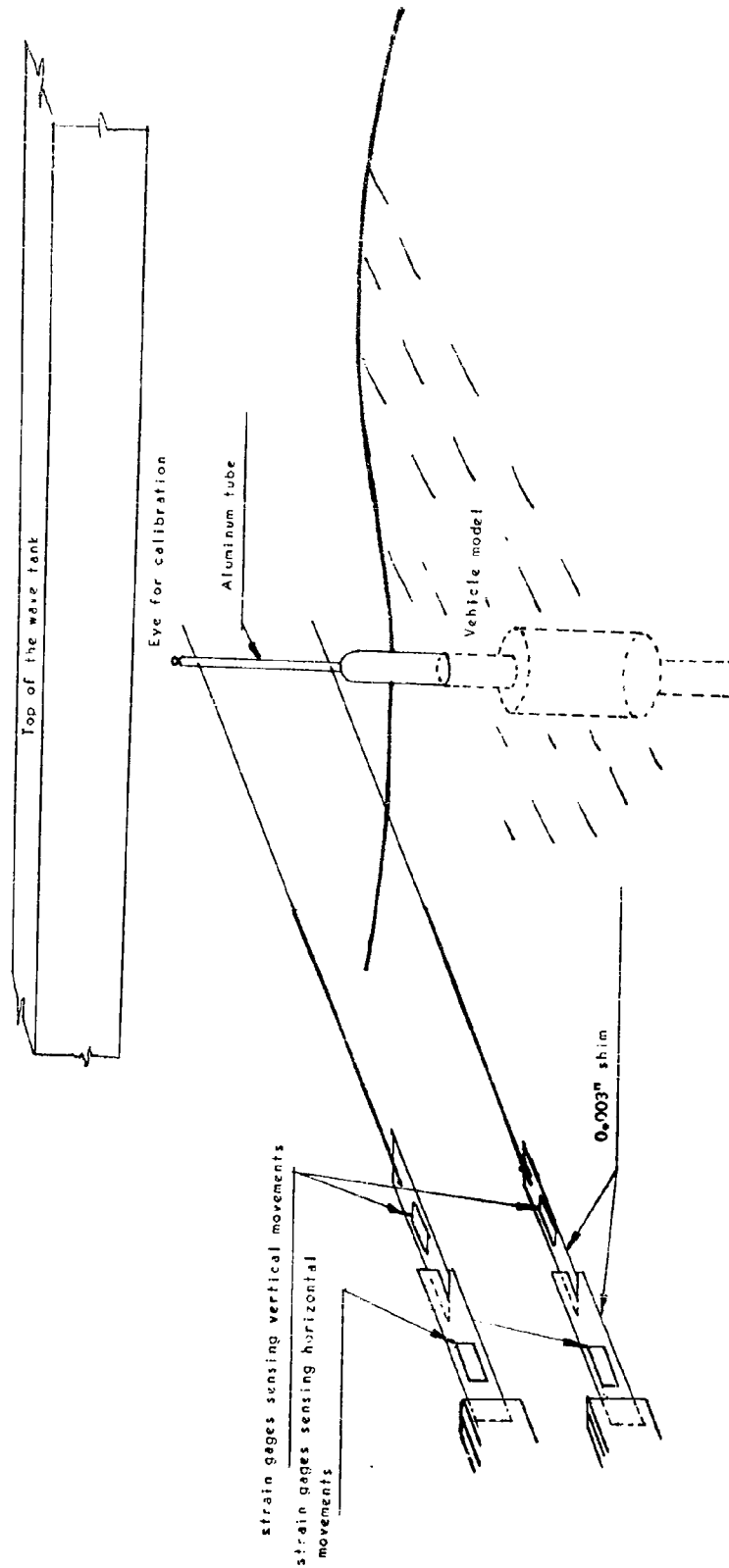


Figure 8. Schematic view of displacement gages.



Figure 9. View of the displacement gages.

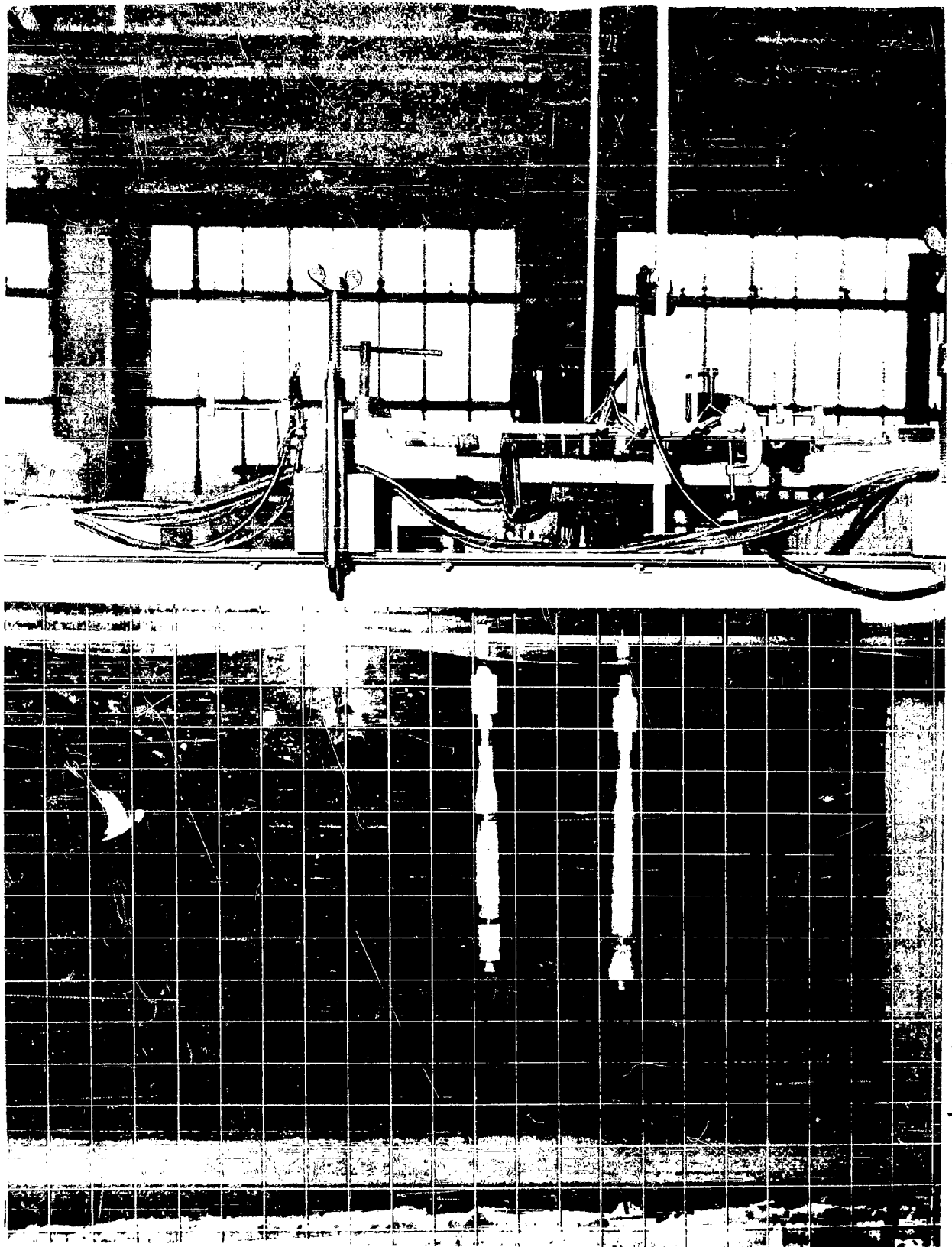


Figure 10. Side view of the tank.

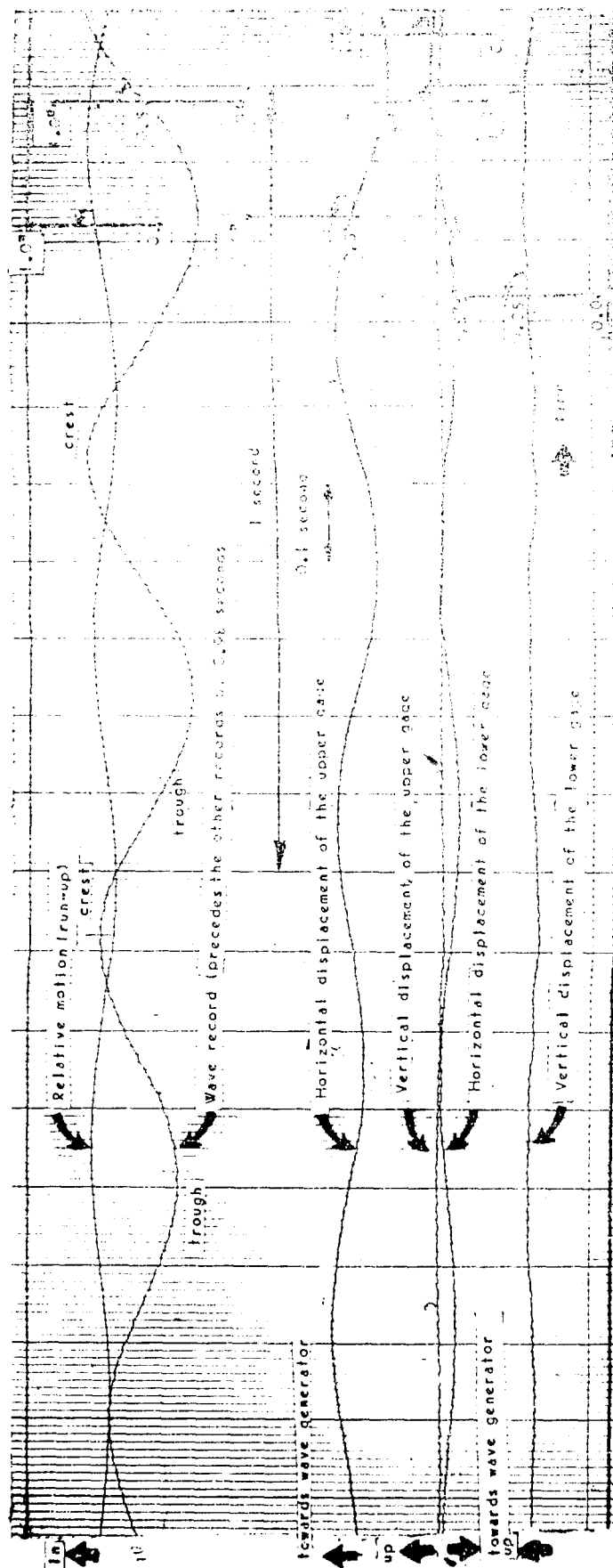


Figure 11. Wave, relative motion, and displacement record.

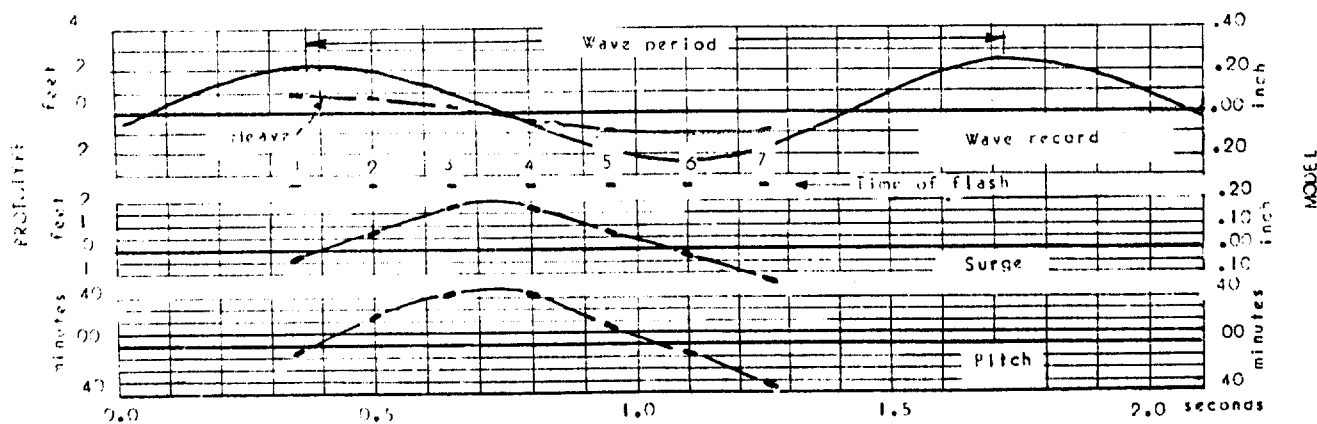
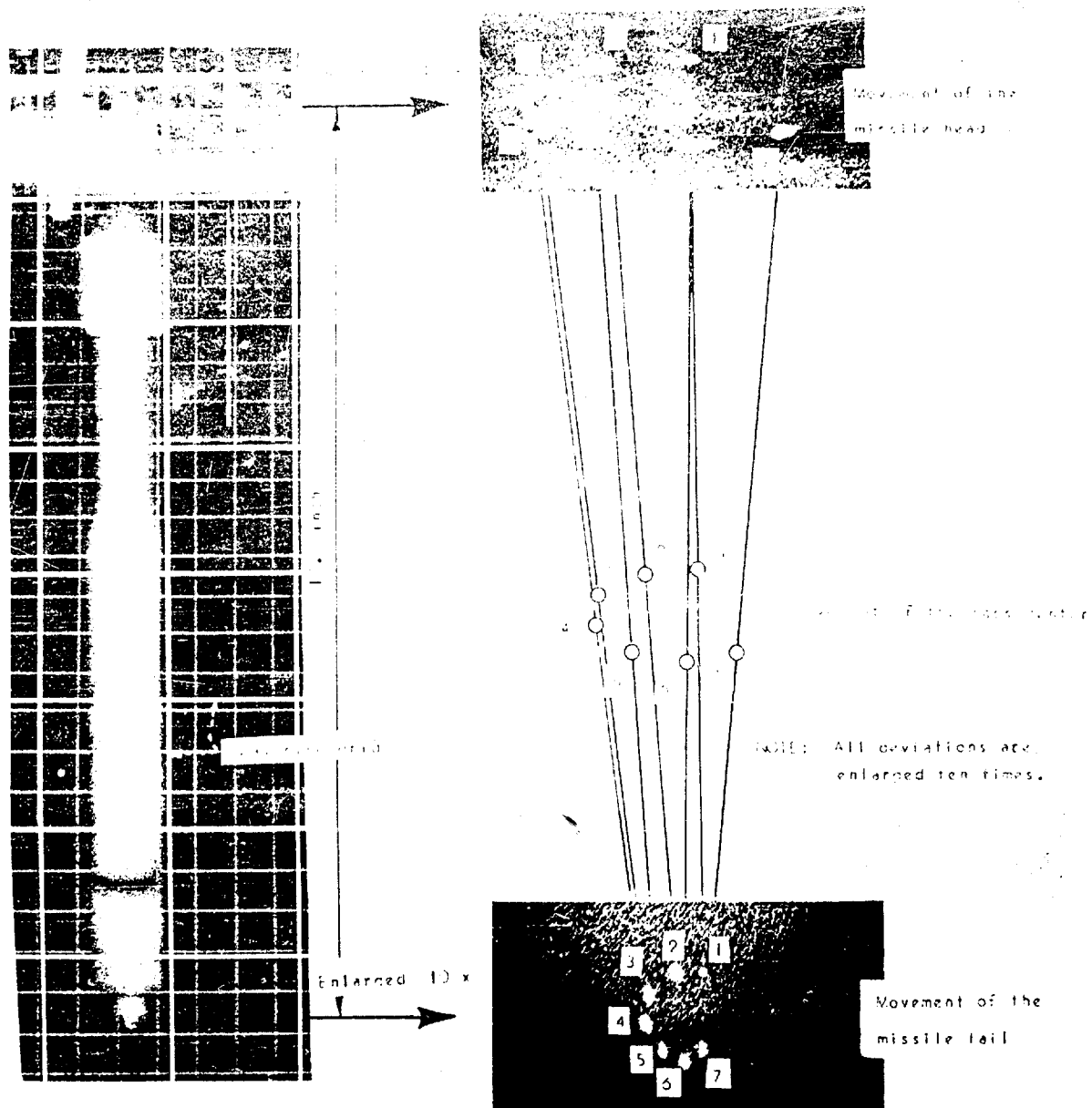


Figure 12. Vehicle model movements. (Model wave height 0.48 in., period 1.34 sec.) (Prototype wave height 4.8 ft, period 14.7 sec.)

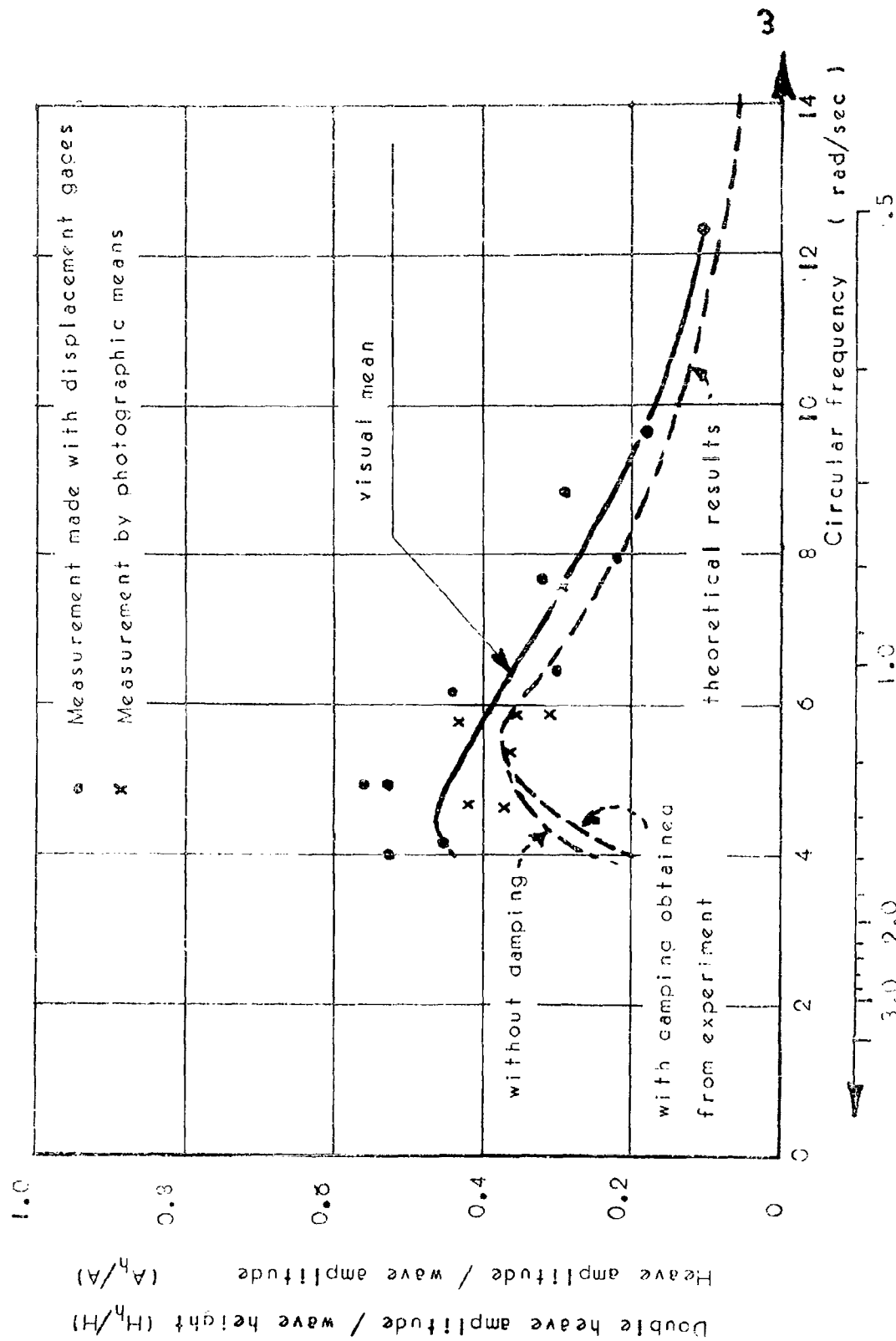


Figure 13. Ratio of heave to wave height as a function of the wave frequency for the model in the wave tank.

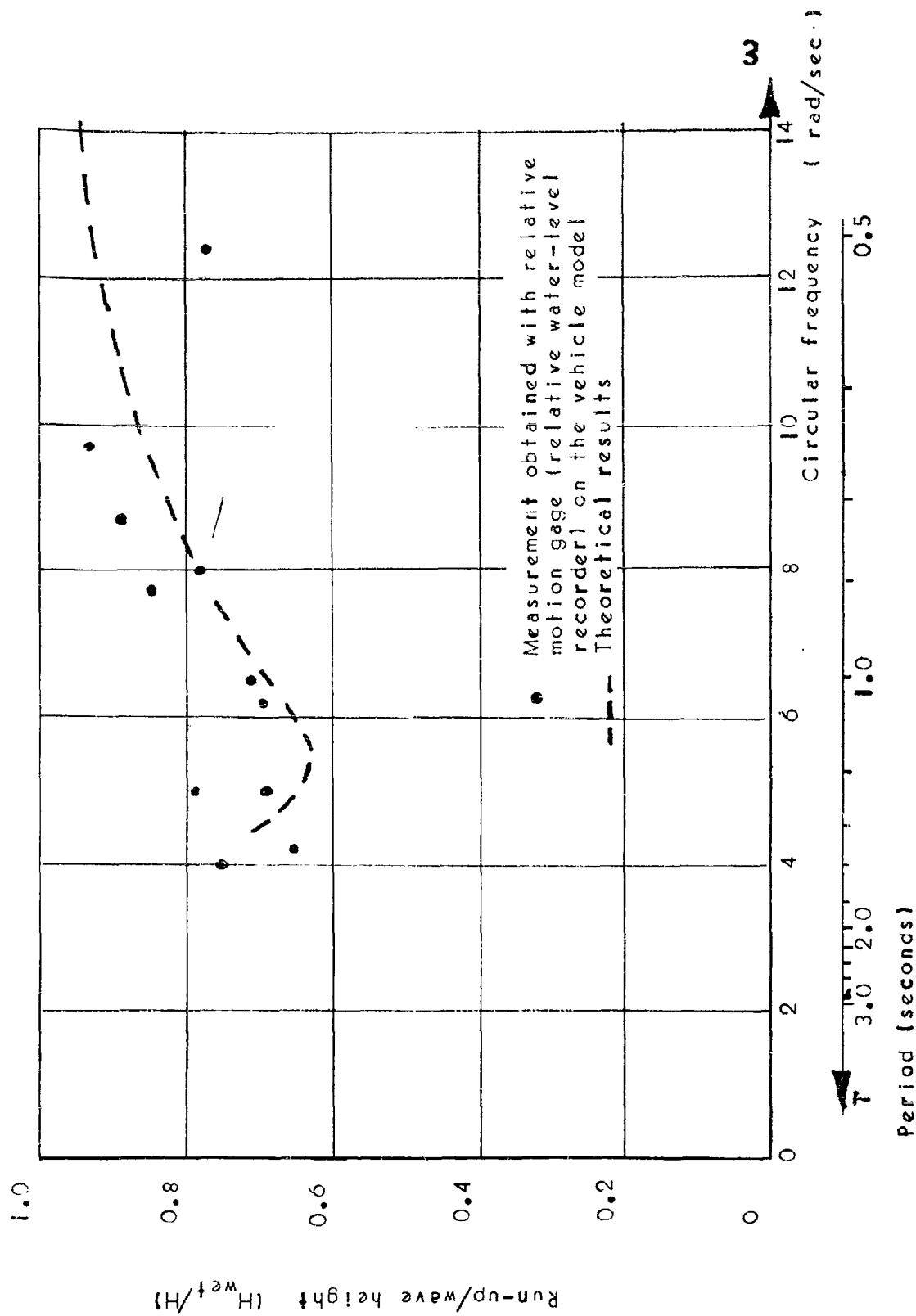


Figure 14. Ratio of wave run-up to wave height as a function of the wave frequency for the model in the wave tank.

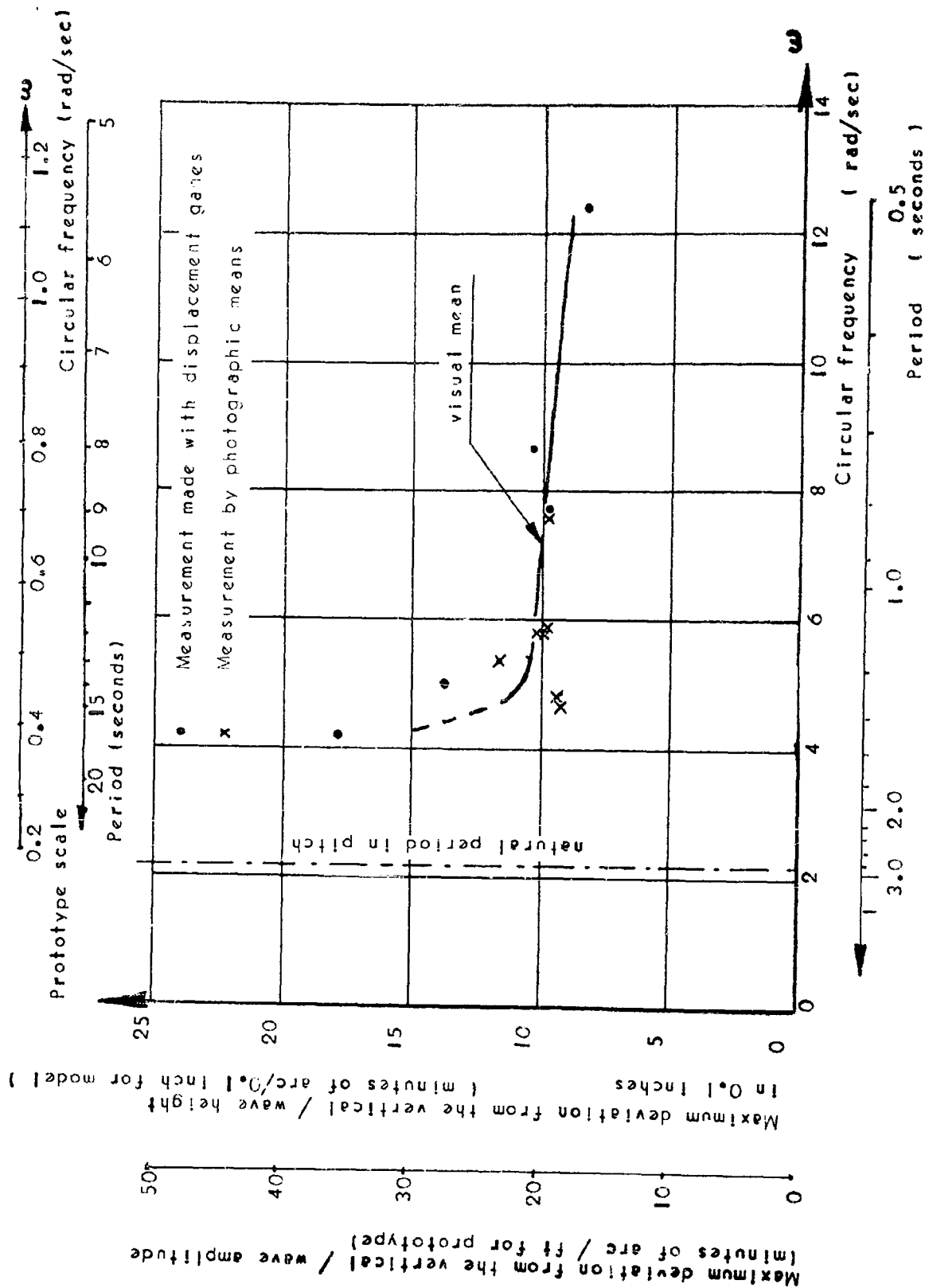


Figure 15. Ratio of maximum deviation from the vertical to wave height as a function of the wave frequency for the model in the wave tank.

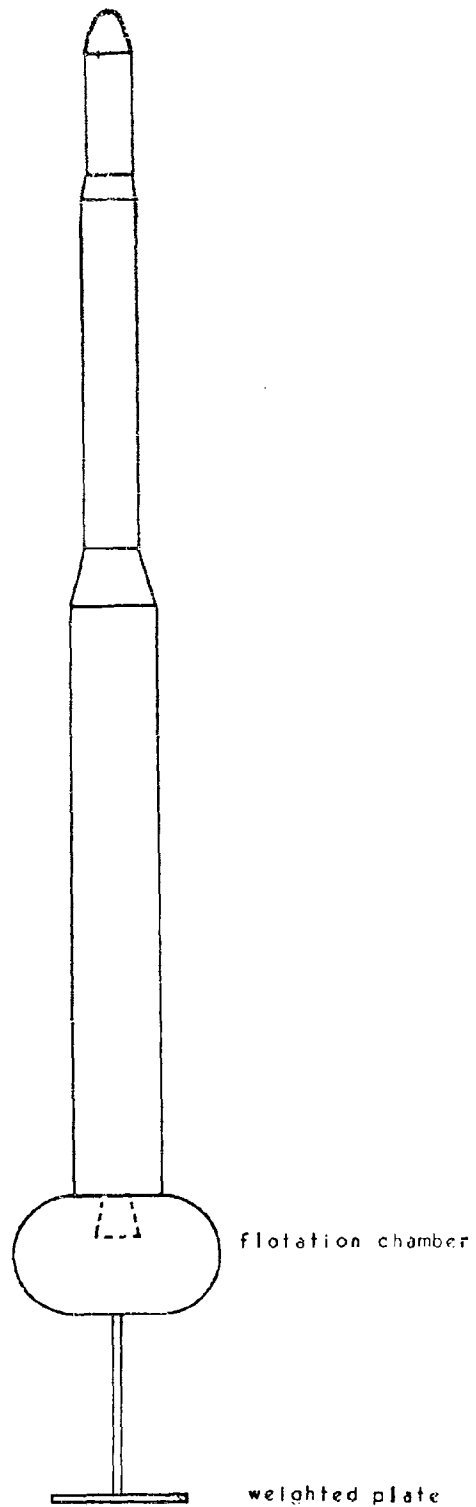


Figure 16. Vehicle model with improved flotation chamber.

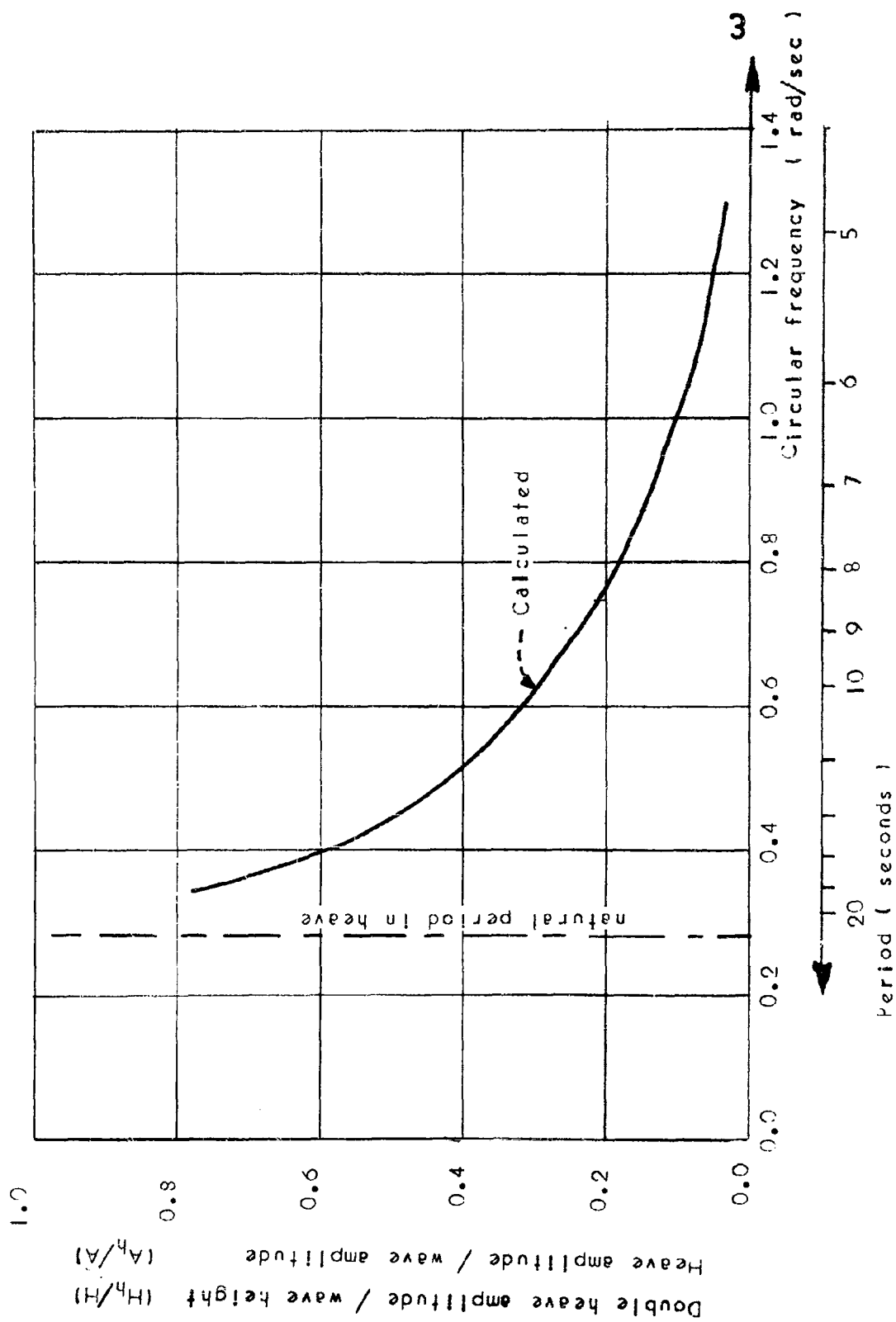


Figure 17. Ratio of heave to wave height as a function of the wave frequency for the vehicle in deep water.

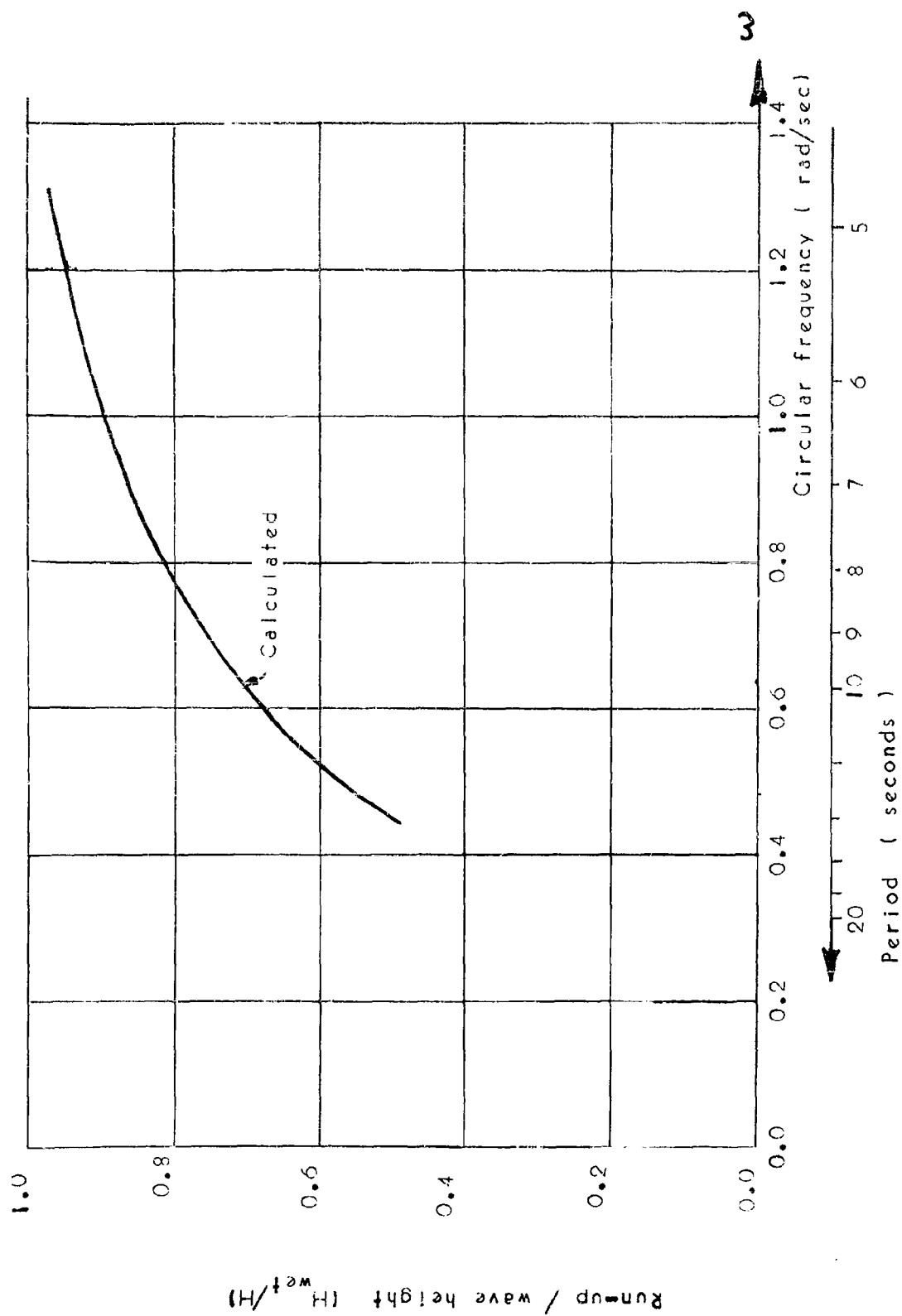


Figure 18. Ratio of run-up to wave height as a function of the wave frequency for the vehicle in deep water.

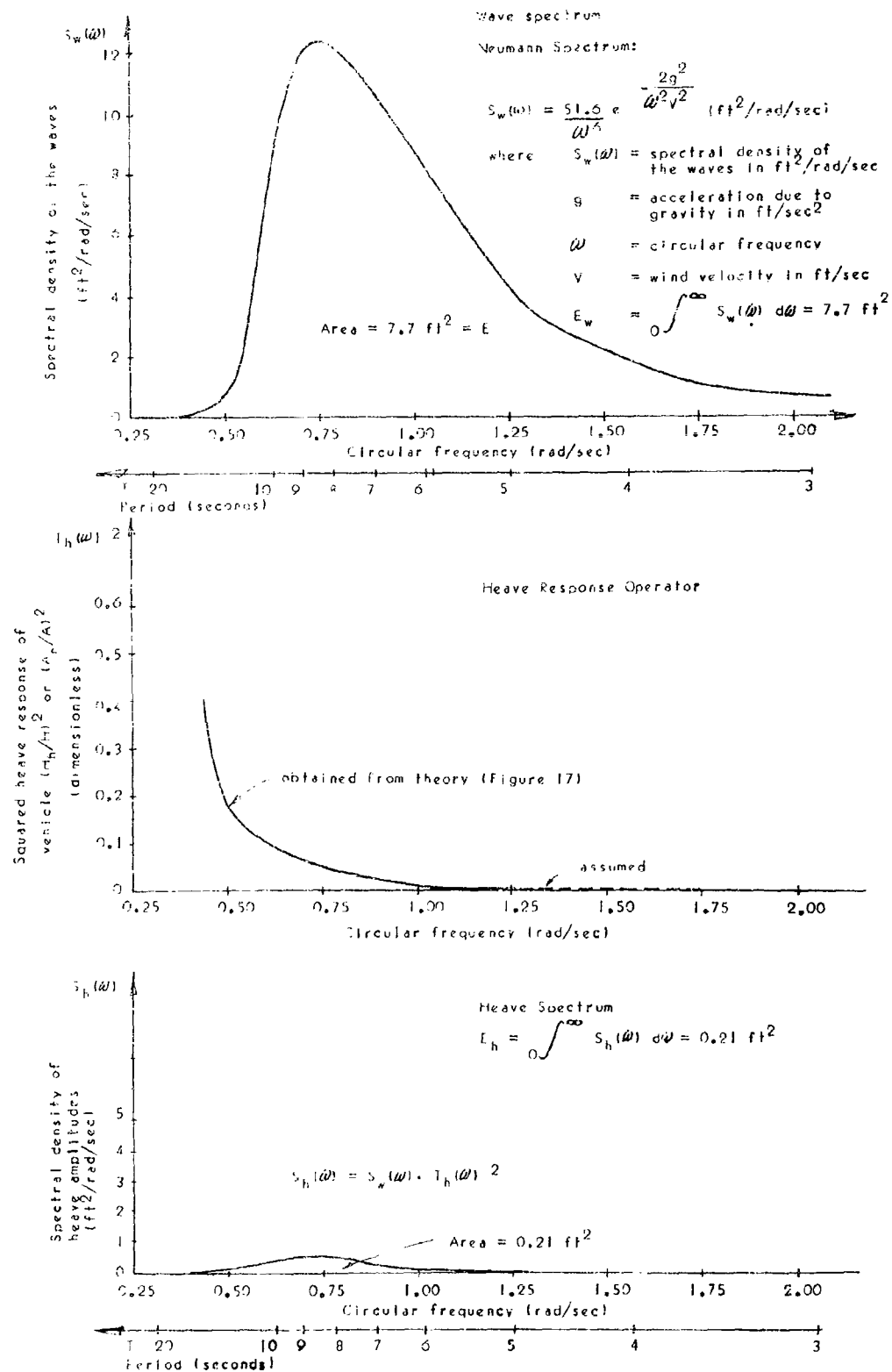


Figure 19. Calculation of heave in a fully arisen sea with a wind speed of 20 knots.

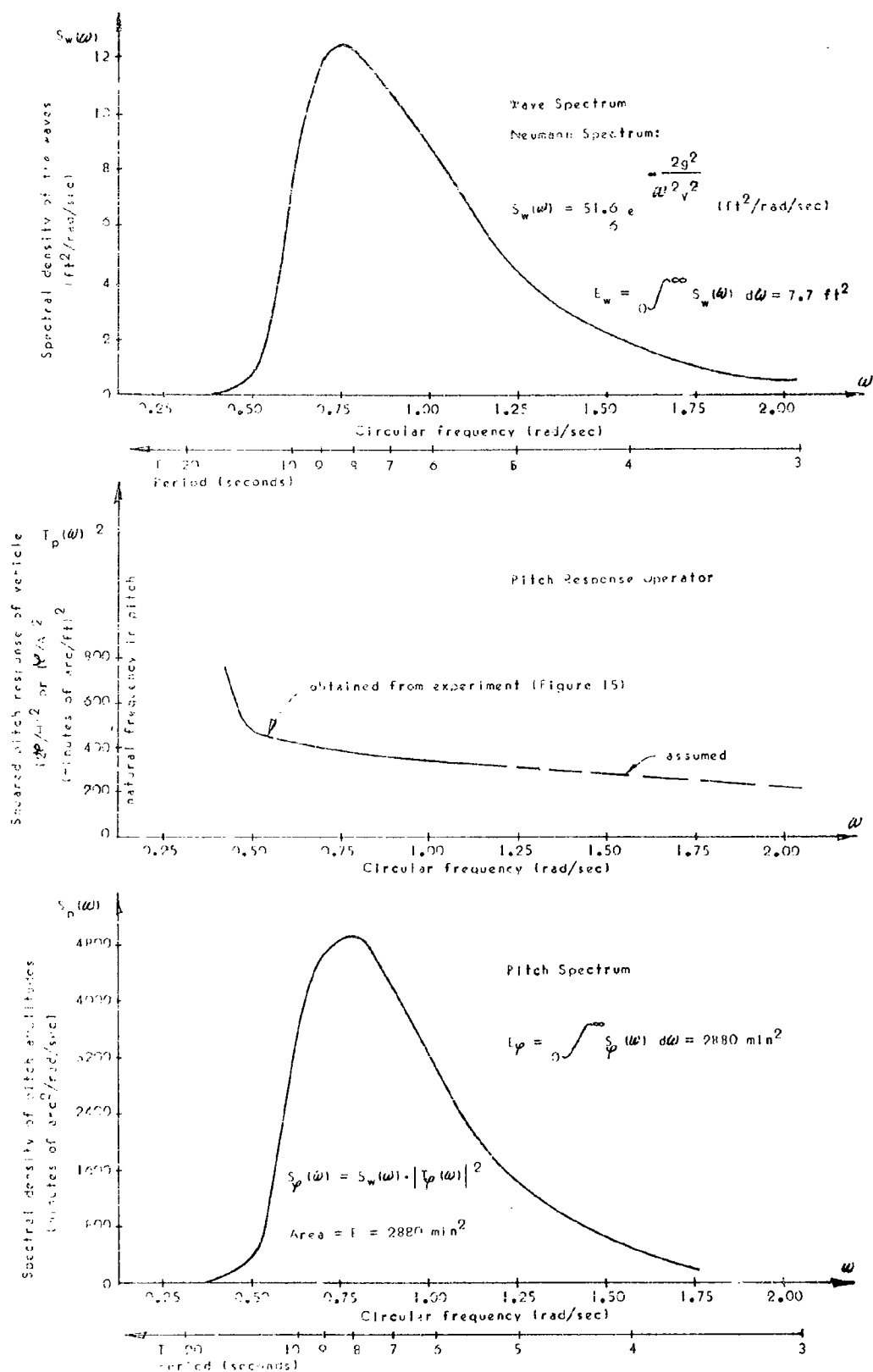


Figure 20. Calculation of pitch in a fully arisen sea with a wind speed of 20 knots.

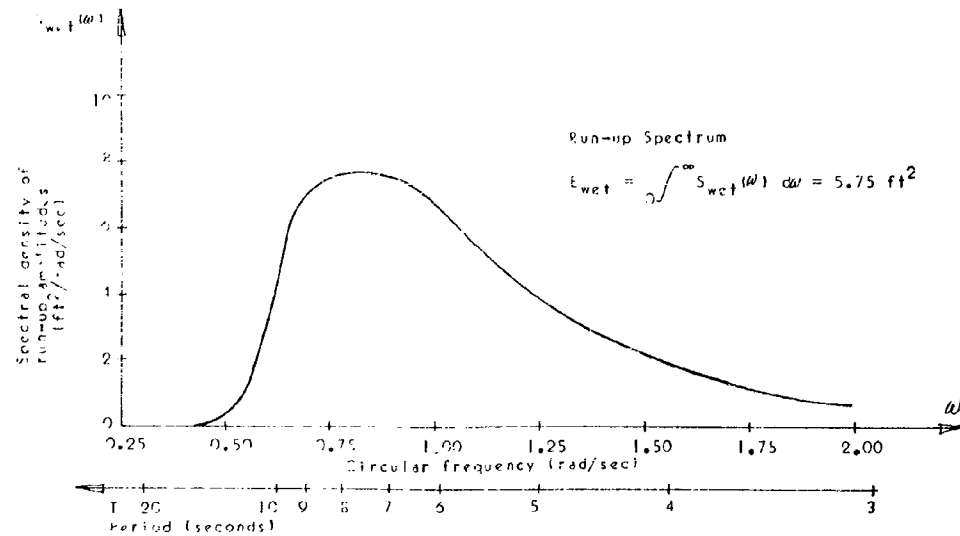
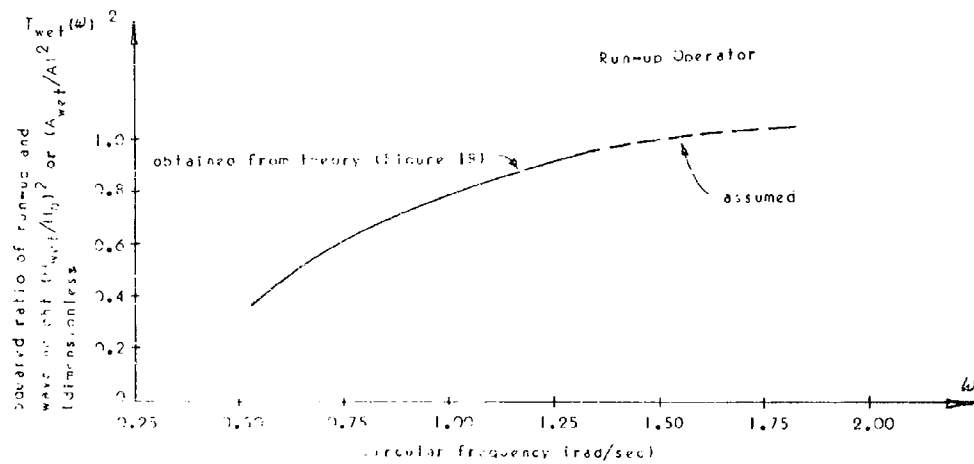
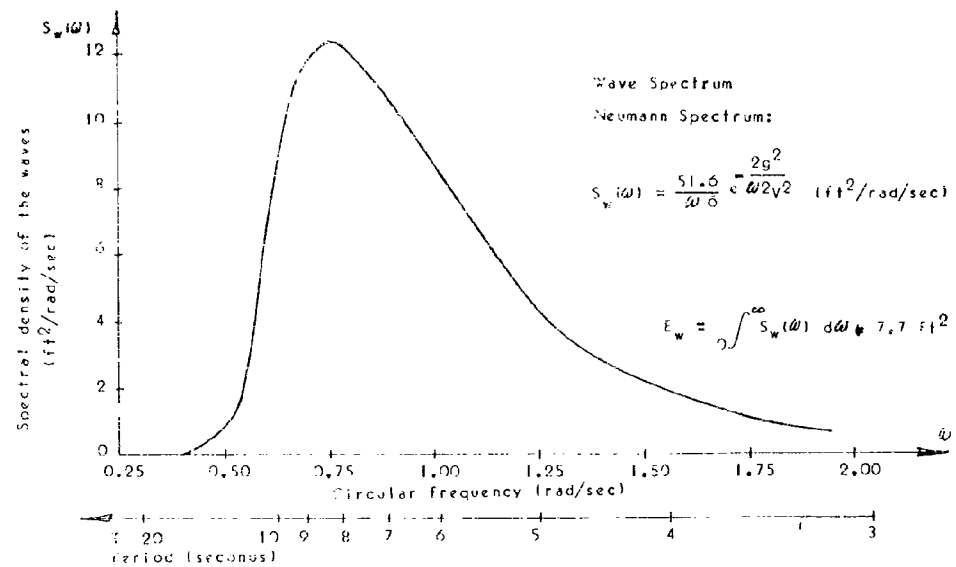


Figure 21. Calculation of run-up in a fully arisen sea with a wind speed of 20 knots.

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NOMENCLATURE

A	= wave amplitude
A_h	= heave amplitude
A_f	= amplitude
A^{resp}	= response amplitude
$[A_h]_t$	= free-heave amplitude at time t
b	= radius of a cylinder
C_d	= drag coefficient
d	= depth
d_n	= depth of considered area under the still-water level
D_n	= distance from the still-water surface to a horizontal plane of the vehicle (Figure 4)
E	= response energy
E'	= mean rate of transmission of energy
E_t	= total mean rate of radiation
f_n	= distance between the mass center of a cylinder and the mass center of the vehicle
F	= significant cross section of the vehicle
F'	= maximum amplitude of the excitation of the analogous system in heave
F_{ex}	= maximum amplitude of the excitation
F_{ex}^h	= total excitation
F_n	= horizontal area
$F_n(p)$	= total horizontal force from the wave pressure on a cylinder
$F_z(p)$	= total vertical upward force from the wave pressures on the horizontal planes of the vehicle
g	= acceleration due to gravity
H	= wave height
H_h	= double heave amplitude

H_w	= run-up
I	= mass inertia moment of the vehicle around its mass center
I''	= added mass inertia moment
k	= $2\pi/\lambda$
L_n	= length of a cylinder
m_ϕ	= metacenter height
M	= mass of the vehicle
M_n	= mass of a cylinder moving horizontally
M''_n	= added mass of a cylinder moving horizontally
M'_p	= maximum amplitude of the excitation in pitch
M''_z	= added mass of the vehicle in heave
n	= cylinder number
N_n	= damping coefficient of a cylinder moving in the X direction
N_z	= damping coefficient in heave
N_r	= damping coefficient due to wave generation
p	= pressure
p_a	= atmospheric pressure
Q	= $\cosh kd (-F_1 \cosh kD_1 + F_2 \cosh kD_2 - F_3 \cosh kD_3 + F_4 \cosh kD_4) - \sinh kd (-F_1 \sinh kD_1 + F_2 \sinh kD_2 - F_3 \sinh kD_3 + F_4 \sinh kD_4)$
R_n	= radius of a cylinder
$S_r(\omega)$	= spectral density at frequency ω of the response
$S_w(\omega)$	= spectral density at frequency ω of the waves
t	= time
T	= wave period
$[T(\omega)]^2$	= square of the ratio of response to wave amplitude at frequency ω of the waves
u	= horizontal velocity of a water particle

\dot{u}	= horizontal acceleration of a water particle
U	= mean horizontal velocity of the water mass displaced by the cylinder
\dot{U}	= mean horizontal acceleration of the water mass displaced by the cylinder
w	= vertical velocity of a water particle
\dot{w}	= vertical acceleration of a water particle
W	= mean vertical velocity of the water mass displaced by the vehicle
\dot{W}	= mean vertical acceleration of the water mass displaced by the vehicle
X	= horizontal displacement of the mass center
\dot{X}	= horizontal velocity of the mass center
\ddot{X}	= horizontal acceleration of the mass center
z	= Z-D
Z	= vertical displacement of the mass center
\dot{Z}	= vertical velocity of the mass center
\ddot{Z}	= vertical acceleration of the mass center
α	= phase angle between wave and excitation
α'	= phase angle between wave and excitation in surge
α''	= phase angle between wave and excitation in pitch
γ	= angle in the horizontal plane
δ	= logarithmic decrement
λ	= wave length
ξ	= phase angle
ρ	= density of water
τ	= time of one complete cycle of the oscillation
ϕ	= angular displacement of the long axis of the vehicle
$\dot{\phi}$	= angular velocity of the long axis of the vehicle
$\ddot{\phi}$	= angular acceleration of the long axis of the vehicle
ω	= frequency of the wave = $2\pi/T$
ω_n	= natural frequency
γ^n	= elevation